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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XLIV Number 7

What Permanent Values Have Courses in Mathematics for Students Who Will Make No Professional Use of Them?*

By NORMAN MILLER
Queen's University, Kingston, Ontario

THE SUBJECT which is broached in my title today is not a new one. Indeed it figures largely, in one guise or another, in the program of this meeting. Further discussion of the topic is suggested by the emphasis which is currently being placed on general education. There are educators who regard mathematics, beyond the elements of arithmetic, as a specialty, and would give it scant space in a general education. On the other hand, this meeting bears witness to the belief of many people that mathematics has *general* as well as *special* values and that its total contribution to a general education is at least as great as that of any other subject. If we hold this belief, our ability to translate our faith into argument convincing to the non-mathematical educators will be the measure of our success in retaining for mathematics its traditional honored place on the curriculum.

I ask you to think, then, of the man or woman who is twenty years out of school or college and has made no specific use of mathematics beyond elementary arithmetic. If there is truth in the old cliché that "education is what remains after all that we have learned is forgotten", then what is the contribution of *mathematics* to education? A metaphor which is apposite if it is not pressed too far is to the effect that it is not the water which remains in the mill which grinds the grain, but the water which passes through the mill. What, in this metaphor, is pertinent to education may be expressed by the statement that a student who has undergone the discipline necessary to master any subject is thereby better equipped for life, even though he forgets the details of the subject—a point of view that used to be insisted on by the late President Lowell of Harvard. As teachers we are naturally anxious to see results of our work and disappointed when these do not measure up to our expectations. We might take comfort from a remark of Sir Richard Livingston of Oxford to the effect that "the most precious fruits of a good teacher's work are those that he is never likely to see".

I shall review briefly what seem to me to be some of the permanent values of the study of mathematics, which justify its being given an important place in general education. By isolating for study the re-

* Read at the 28th Annual Meeting of the National Council of Teachers of Mathematics, Chicago, Illinois, April 14, 1950.

sults which should be achieved, we may throw some light on teaching methods by which we may more effectively achieve them.

Training in Language

It is my belief that a training in the exact and effective use of one's mother tongue should be one of the important permanent values of the study of mathematics. To realize this result requires a clear objective and no little patience on the part of the teacher. He should require that a pupil should at all times say exactly what he means, with an economy of words but with no fuzziness or indefiniteness of meaning. He should insist relentlessly that a student should never equate two things which are not equal—an angle to its sine or a number to its logarithm. How much clarity would be gained in the thinking and speaking of everyone if they never said or implied the equivalence of two things which are demonstrably not equivalent!

In a mathematical argument one's mind may take leaps. A good student often sees the conclusion before you reach it. But it is an important part of his training to marshal his arguments in orderly fashion so that there are no gaps and every step carries conviction. This habit, if insisted upon, is surely one that should remain a permanent possession of an adult after he has forgotten the rules of logarithms or the formulas of trigonometry.

Language and thought are so closely interdependent that one can hardly imagine improvement in one without a corresponding improvement in the other. Careful teaching in mathematics should make for economy and conciseness and against prolixity in the use of words. When I meet an adult who has a greater than usual power in the spoken and written word, the conclusion always leaps to my mind, "He must have been a good student of mathematics." Again, when I hear a preacher or other speaker who wraps up a subject in an endless proliferation of words, some of them inept and inappropriate, I groan: "He should have paid attention to mathematics." In Canadian parliamentary life some three decades ago there was a particularly keen debater who was known for the effectiveness and incisiveness of his language. It did not surprise me to learn that as an undergraduate he had majored in mathematics.

The objection may be raised that, if improvement in speaking and writing is desired, more time should be spent in formal instruction in English, at the expense of other subjects. The answer is two-fold: 1) English is not the only subject which is benefiting in mathematics classes; indeed the improvement in English is a by-product; 2) The learning of a subject is often more effective when it is shown to have meaning and use in other subjects. I recall one discussion of suggested curriculum changes in which a teacher of English vigorously objected to the large amount of time which it was proposed to devote to English. His argument was that English composition should be the concern of all teachers and that to devote an undue amount of time to formal classes in English was to invite other teachers to reject any responsibility in the matter.

My final remark on this subject of language must be that if worthwhile results are to be achieved they must be worked for. It is a temptation for a teacher, always faced with the spectre of a final examination, to concentrate on the facts and formulas of the course and to pass over inaccuracies of statement with the remark, "I see what you mean", or "You have the right idea but you haven't expressed it very well". I urge you to withstand the temptation, to take time off until the pupil has expressed his idea well, until he not only has the correct idea in his mind but also the correct expression for it in words. Let us remember that these facts and formulas will be forgotten in five or ten years but they will have served

their purpose if they have been the means of inculcating a habit of meticulous accuracy in the use of words.

Training in Reasoning

I have mentioned that improvement in one's reasoning power and in his ability to express his thoughts go hand in hand. It is trite to remind you of the age-old claim that it is in mathematics that pupils meet their first and clearest example of a logical argument. It is one of the most satisfying experiences in a child's intellectual growth when he acquires the conviction that some things have to be accepted and believed, not because his parents or his Sunday-school teachers say so but because he feels that the conclusion is inevitable. His ability to reach a conclusion by carrying through an argument requiring two, three, or more steps is a measure of his intellectual growth.

The implication of this for us teachers of mathematics is that, for the purpose of general education, it is not the conclusions in themselves that are the important things. To paraphrase a well-known remark of Robert Louis Stevenson, the exercise which we get in the course of our journey is a more important objective than the destination which we reach at the end of it. We should not be in too much of a hurry to reach the conclusion that the angle in a semicircle is a right angle or that a polynomial in x is divisible by $x-1$ if the sum of its coefficients is zero. If a student memorizes the statements and even the proofs of all the theorems in the text-book, but does not acquire an ingrained feeling of the inevitability of the arguments which led to these theorems, he will have little of his mathematics course left after twenty years. He will forget the theorems but, with proper teaching, his mathematics courses should make a difference to the way his mind reacts to the problems which he will meet in his family life, in business, in politics, and in religion.

Another implication of which teachers are becoming more and more conscious is that the traditional material of geometry can usefully be enlarged by the introduction of other situations related to the pupil's experience, on which his reasoning powers can be exercised. Geometry is the closest approach that most pupils will ever get to the study of formal logic. Indeed some formal logic may find its way into school courses in geometry.

I realize that in urging as a permanent value of a training in mathematics the improvement in one's reasoning power I am treading on the fringe of the controversy on the transfer of training. I suggest to you that teachers of mathematics with vision and enthusiasm have never been in doubt on this question.

Training in Historical Perspective

Some educators are in the habit of dividing academic subjects into two classes one of which, labelled the humanities, is supposed to have the greater value for the development of the student's personality, his outlook on life, and his feeling for his fellowmen. In the second category of inhumanities are grouped mathematics and science. I do not see much value in this classification. All depends on how a subject is taught. If such a division must be made, then mathematics has a good claim to be classed with the humanities—as the Greeks and Romans did in their quadrivium of arithmetic, geometry, astronomy, and music. I am not alone in making this claim. According to Sarton, the historian of science, the history of mathematics should really be the kernel of the history of culture.

If we make such a claim for mathematics, then it is for us to substantiate the claim in our teaching. This point of view was developed in an address in 1912 by Professor C. J. Keyser on The Humanization of the Teaching of Mathematics. "It is sufficient and it is necessary", he urged, "greatly to enlarge, to enrich, and to vitalize our sense of what mathematics, regarded as human enterprise, signifies."

The more we illuminate our subject with historical or inspirational material, the greater value will it have as a part of general education and the more will it contribute to the student's adult mental equipment.

One way of enriching our teaching is by placing ideas and discoveries in their historical perspective. Since much of high school mathematics originated with the Greeks, it is possible to give the student an appreciation of and admiration for the accomplishment of this ancient people, pointing out the handicap under which they worked on account of the lack of a supple algebraic notation and the clear-sightedness of their reasoning about things which they could visualize. Euclid's proof that the number of primes is infinite, Pythagoras's proof that the diagonal of a unit square is measured by no rational number, Hero's formula for the area of a triangle—these are achievements which can hardly fail to kindle the imagination of a student and possibly to stimulate him to further reading of his own.

To take a leap of twenty centuries, we may mention another outstanding period in the history of mathematics—the seventeenth century. This is the century which saw the invention of logarithms, of analytic geometry, and of calculus. A student who reaches these subjects in his study of mathematics will acquire a deeper sense of the significance of this century than he will by a study of its political history.

Let me illustrate this point. When logarithms have been taught and when the students realize that their power arises from a judicious use of indices, it is illuminating to point out that Napier, their inventor, made no use of indices; indeed indices came into use after Napier's time. If this information prompts questions from a few eager students as to how this was possible, they may be directed to the address of Lord Moulton in the Napier Memorial Volume. The student who has thought about this question and read this essay will have an appreciation of greatness in a human being which will not depart from him when he forgets the rules of logarithms.

A greater opportunity for the humanizing of our subject is furnished by the introduction of calculus. Why did a new and revolutionary method in mathematics come into being in the seventeenth century? Why were the methods which served Archimedes and Apollonius, or at least those which served Fermat and Descartes, not sufficient for Newton? In answer to these questions, one may point to the rebirth of learning in Europe which, beginning in the late fifteenth century, stimulated literature first and science somewhat later. By the middle of the seventeenth century a great deal of information in science had been accumulated and the subject was in a healthy ferment. The names of Galileo, Tycho Brahe, Kepler, Descartes, Pascal, Harvey, Napier, are enough to suggest the discoveries which were being made in various countries of Europe. What kind of world did these discoveries reveal? Borrowing a word from Newton, we may call it a world in a state of flux. The science of dynamics stems from Galileo and the early seventeenth century. Now the mathematics of the Greeks and of the Schoolmen in the Middle Ages was good enough to describe a static world, a world at rest,—good enough for the moon, but not for the earth. For ours is a changing world, a dynamic world, a world in a state of flux. The phenomena in which Newton was interested were winds and tides, the motion of the moon and of the planets, gravitation, light, chemical changes, and for these he found the older mathematics inadequate. Mathematics for a world in motion, a world of change, a dynamic world, a world of variables,—this is the essence of the calculus. A student to whom this has been pointed out, at greater length than I have time for here, will retain an appreciation of one of the greatest mathematicians, Sir Isaac New-

ton, and of the time in which he lived, which will remain when he is no longer able to carry out the differentiation of the sine of x. Moreover, a teacher who fails to take advantage of this humanizing possibility of his subject and introduces the calculus by plunging at once into the formal process of differentiation is neglecting a valuable opportunity of general education.

Let us continually vitalize our teaching from the history of mathematics and from the biographies of great mathematicians. By imparting, along with the technique of mathematics, an appreciation of the great periods of discovery in the history of thought, we establish mathematics as one of the greatest means of a permanent and humanizing education and repudiate the term *dry-as-dust* which should have no right whatever to be associated with mathematics.

The Applicability of Mathematics

In speaking of applications of mathematics, I am not thinking here of engineering or statistics or actuarial work. There is no question of the necessity of mathematics for these fields. The pupils we have in mind are those who will make no professional use of the mathematics they learn. I take my cue from a remark of Henry Adams in his autobiography, *The Education of Henry Adams*. "Nothing in education," he says, "is so astonishing as the amount of ignorance it accumulates in the form of inert facts." By what precautions can we ensure that mathematics does not become for our pupils an accumulation of inert facts? I have already mentioned some of these. It remains to emphasize that what remains in memory after twenty years is largely a function of the connections that have been established between the mathematics he learns and the experience of the learner, whether this experience is acquired either in or out of school.

One implication of this is that connections are to be sought between topics in different subjects. The fact that a pupil is likely to receive a stronger impression from what he sees than from what he hears is the justification for the use of graphical illustrations, wherever possible, in arithmetic and algebra. A graphical study of the quadratic function gives reality to the algebra of quadratics. A graphical or experimental demonstration of the resultant of two forces is not likely to be forgotten and the same applies to the principle of the lever, the working of pulleys, and the effect of friction. The wind-rose of the meteorologist and his barometric pressure charts showing *low* and *high* areas are further examples of graphical aids. Complementary to the use of graphs in the study of algebra is the fact that algebraic proofs of geometric theorems serve again to tie the two subjects together and to provide suggestions to the student in his search for solutions of his own problems.

Opportunities are to be sought also to connect mathematics with the daily life of the pupils. For those who come from farms one can use examples involving such items as troughs and tanks, feeding mixtures, fertilizer ratios, and butterfat content of milk. For city pupils examples are readily available in buildings and machinery, budgets, and cost-of-living index numbers. The seventeenth yearbook of the National Council is a useful source of material for teachers who do not make mathematics a collection of inert facts. Not only do such applications impress the meaning of the subject on the pupil, but they also provide the basis for an appreciation of the scientific age in which we live.

One great teacher who stressed the importance of applications was Felix Klein. He was, moreover, wise enough not to exalt them to the exclusion of the main aim of mathematics. His advice still has point for us: "We want a quickening of mathematical instruction by the introduction of applications but we do not wish that the pendulum, which in former decades may have inclined too much toward

the abstract side, should now swing to the other extreme; we should rather pursue a proper middle course."

The teacher who has his eye on long range values of his subject will strive for concreteness in his own language. The teacher's experience and the width of his reading show up in the language of his teaching. Along with concreteness and contributing to it, let me mention the saving grace of humor which is as wholesome in the teaching of mathematics as in other situations. Lewis Carroll was a mathematician as well as a writer of delightful fiction. Teachers of mathematics today will probably profit more from his fiction than from his mathematics. I take one example. Mathematicians have the practice, in naming mathematical concepts, of using common words and giving them definite meanings which do not agree precisely with their meanings in common use—such words as *real, imaginary, function, set, group, ring, field.* What more apt comment on this practice than the statement of Humpty Dumpty to Alice in Wonderland regarding his use of words: "I pay them extra and make them mean what I like"!

The Personality of the Teacher

You cannot have failed to notice how often I have mentioned the role of the teacher. I shall close on this note.

Felix Klein, whom I have already mentioned, is responsible for the statement that teaching is a function of two variables, the subject taught and the pupil. This dictum implies an ideal teacher. In this imperfect world I would add a third variable—the teacher. What does the pupil remember from his mathematics courses after twenty years? One answer, whether we like it or not: he remembers us, his teachers. How often does it happen that, when adults reminisce about their school days, their talk is mainly of their teachers! How important, then, that the impression made by the teacher's personality should be such as to emphasize the importance and dignity of his subject and its value as a part of our cultural heritage! Whether the residue in the adult's mind from his study of mathematics has associations which are pleasant or unpleasant, inspiring or dull, depends as much as anything on the personality of the teacher.

I have just referred to one great teacher. I refer now to another. When his pupil, anxious to arrive at the value of what he was learning, demanded: "What do I get from learning this?", the teacher told a servant: "Give him a penny, since he must make a profit from what he learns." There is a moral to this story but I have never thought that this impatient answer was given in one of Euclid's most inspired moments. A greater teacher than Euclid gave a more illuminating answer. His disciple also was in a hurry to fix upon the core of his teaching and demanded: "Show us the Father and it sufficeth us." You remember the answer; it illustrates my point about the importance of the teacher and his personality. The answer was: "Have *I* been so long time with you, and have you not seen the Father?"

The "CARE-for-Korea" campaign voted by the General Federation of Women's Clubs at Houston will open on November 12th and end on Thanksgiving Day—in time to save Korean women and children from hunger and cold this winter. CARE's package program for Korea has been approved by the U. S. State Department's Advisory Committee on Voluntary Foreign Aid and has been endorsed by President Truman and General Ridgway. Funds raised in the campaign will be applied toward six types of CARE packages compiled to meet specific Korean needs: woolen blanket, cotton textiles, $7 each; food, woolen fabric, underwear, $10 each; knitting wool, $13. Delivery will be made to the most destitute orphanages, refugee families and other civilian war victims.

Flying Saucers—A Project in Circles

By NINA OLIVER
Ardmore High School, Ardmore, Oklahoma

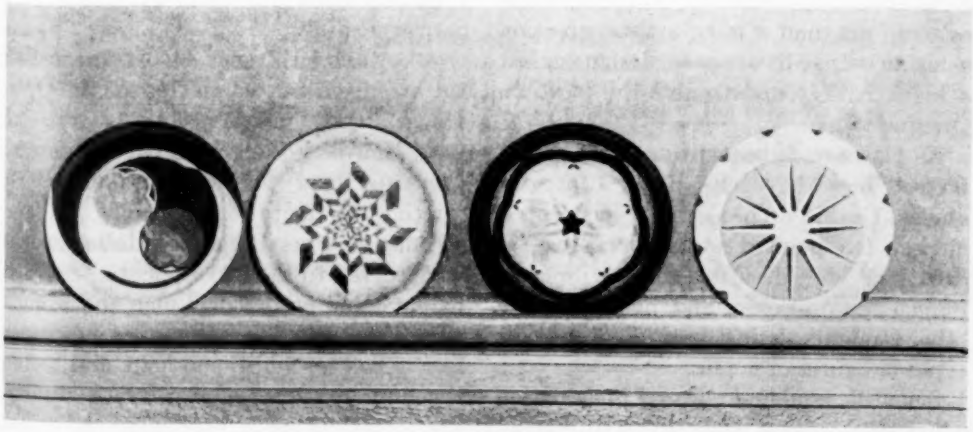
"YOU are now entering the Hall of Geometrical Flying Saucers. Famous stars, well-known squares, and scandalous triangles dwell here. . . ." This announcement, written by a plane geometry student with a flair for the theatrical, topped ballots handed to guests entering the Open House exhibit of our Flying Saucers project in circles.

Until we began the project in our three plane geometry classes at Ardmore High School, geometry theorems on determining the center of a circle, on central angles and their arcs, and on regular inscribed polygons were just another routine. Now it's a different story, as students thrill to the realization that geometry moves in the best of circles, compatible with art, design, architecture, and other professional fields.

As my plane geometry classes were approaching the study of circles this year, I began searching for a new idea to vitalize the subject. After using compasses and straight edge for several months, we started preliminary work on vocabulary by discussing construction of windows, arches, and columns of some of the attractive church buildings in our town. To transfer such structural ideas over into the classroom and to experiment mathematically and artistically, we evolved a unique plan. We decided to use paper pie plates of exactly the same size, creating designs for these plates by applying theorems of geometry.

Each student chose his own medium to use—water color, oil paint, ink, poster paint, airplane dope, enamel, shellac, pastel, poster-pastellos, crayolas, charcoal, or embroidery. He was also free to inscribe any type of polygon, arc, sector, star, etc., but had to be able to explain how his work was constructed. Students developed their own designs and color schemes and, when all the plates were in, we had a most attractive Flying Saucers display. Even some of the slower geometry students had top rating designs in superb color combinations. The designs ranged from extreme formal patterns to freedom of modern art and architectural designs.

This six-week project, to be done in marginal time, was required of every student. When designs arrived, they were put on display on the plate-rail above the



Detail of several plates entered in the Flying Saucers Contest

Students of Ardmore High School holding their winning designs are from left to right: Marie Edens, Don Butcher (who won both second and eighth places), Harold Burton, Mary Hunt, Jimmi Shortes, Fanelia Chadwell, Joe Gillian, and Max Montgomery. Mrs. Nina Oliver is holding the plate of one of the winners who was absent at the time this picture was taken.

blackboard. It became a matter of daily suspense to see just what new pattern each day would bring. Students were eager to explain their creation, and to tell how they had used training in art, mechanical drawing, home economics, or science in completing projects.

One student who is especially interested in atomic studies had the nucleus of an atom centering his plate, with spherical circles illustrating the protons, and electrons revolving around it. Another, interested in art, had a most artistic creation using an orange flower-petal design against a black background to give the plate an Oriental touch.

As plates were brought in, students explained how they constructed 72 degree angles, Gothic arches, spirals, concentric circles, five-pointed stars, octagons, or decagons. They discussed reasons for combining certain colors and lines, warning other students against hazards of difficult types of figures or media.

Scientific students released imagination concerning atomic structure and fission, appreciation of light spectrum and the color circle, and knowledge of the points of the compass. One young philosopher showed mankind pondering the fate of civilization in the atomic age, with a prediction of a happy outcome. His plate showed The Thinker with an atomic cloud hovering overhead, and flame edging an inner circle just inside the green outer edge of the plate. Individuality was portrayed by each design, and at the conclusion of the six-week period we were very proud to open our room for the Flying Saucers exhibit.

A boy and girl from each class served on the arrangements committee for the Open House. They made posters advertising the "Hall of Flying Saucers," wrote and presented over the loud-speaker a skit inviting students and faculty to the "Flying Saucers Premier." Refreshments were served to the 500 students, teachers, parents, and professional men and women who attended. The committee in charge had to call for additional help, and before the day was over almost every student had served as guide, explaining the geometry and medium of the various saucers.

Each guest was given a ballot, and was asked to vote on the ten plates he liked best. In addition to the blurb already quoted, guests were admonished to "please examine our space-ships carefully and vote for the ten that fly the highest with you." Among parents and professional people who came to visit and to vote were two outstanding architects, an art teacher, the high school principal, and the superintendent of schools.

The students were proud of their creations, and when the project was completed they felt they had gained a new conception of the meaning of geometry. Now they see more than circles and angles and theorems. They see cathedral windows, interior decorations, patterns for printed materials, designs for furniture, and a whole group of other possibilities still unexplored.

Texas' First Mathematics Institute was held on the campus of the University of Houston from July 30 through August 2 and was sponsored by the College of Education, the Mathematics Department and the College of Business Administration of the University. The purpose of the Institute was to stimulate interest in the teaching of mathematics, to show its practicality as a subject in the curriculum by showing its uses in business and industry and to help teachers of the subject to acquire ideas as to the improvement of their teaching.

Dr. William A. Brownell, Dean of the College of Education, University of California at Berkeley, was the guest speaker and consultant in the field of arithmetic. Dr. E. H. C. Hildebrandt, Professor of Mathematics at Northwestern University, Evanston, Illinois, served in the same capacity in the field of secondary mathematics teaching, and Dr. E. T. Bell, Professor of Mathematics at the California Institute of Technology, Pasadena, contributed in the area of college mathematics. These authorities in their fields greatly stimulated the thinking of all who participated in the Institute.

An outstanding feature of the Institute was a series of study groups held each afternoon where classroom teachers could gather and discuss down-to-earth problems that occur in their daily teaching. Eight of such groups were conducted under the leadership of Texas people, including Miss Cecile Foerster, Supervisor of Kindergarten, First and Second Grades, Houston Public Schools; A. E. Wells, Superintendent of Schools, Abilene, Texas; Miss Lillian Smythe, Sixth Grade Teacher, Pasadena Public Schools; Dr. I. I. Nelson, Professor of Curriculum and Instruction, University of Texas, Austin; Dr. Ruth Lane, Associate Professor of Mathematics, Sam Houston State Teachers College, Huntsville; Miss Ruby K. Jones, Mathematics Teacher, Odessa, Texas; Dr. C. O. Mitchell, Chairman, Business Administration Department, East Texas State College; Dr. W. I. Layton, Head of Mathematics Department, Stephen F. Austin State College; and Dr. W. L. Porter, Head of the Mathematics Department, A. and M. College at College Station.

The Institute was opened by an address by Dr. M. M. Slotnick, Chief Mathematician of the Humble Oil and Refining Co. of Houston, who spoke on "The Place of Mathematics in Industry." It was interesting to know that mathematics occupied such an important position in all aspects of business, industry, and life in general.

The Institute did not neglect the social aspects of living either. Mathematics teachers enjoy human relationships as do all people. A luncheon was held on Monday at which people became acquainted and mingled socially. That evening a social hour featured "The Singing Hicks," and square dance demonstrations. On Wednesday a banquet was held in the ballroom of Oberholtzer Hall. The speaker for the evening was George Carmack, Editor of The Houston Press. Under the direction of Dr. C. F. Hiller, Vice-President in Charge of University Development and Public Relations, who acted as Master of Ceremonies, the affair provided humor, stimulation, and relaxation as a fitting climax for the Institute.

Attendance was not confined to local people. Three hundred twelve people registered, some coming from Arkansas, Louisiana, West Virginia and Ohio.

Proceedings of the Institute were published early in September. Any one interested in obtaining a copy may send his order with check or money order for $1.50 to C. B. Rader, Mathematics Department, University of Houston, Houston 4, Texas.

ESTHER F. GIBNEY

The National Council welcomes invitations from groups willing to serve as hosts to any of its meetings, and where the facilities are adequate. If your organization will sponsor a meeting, send your invitation to the President for consideration and action of the Board.

Arithmetic for Kindergarteners

By LORETTA U. BOYCE
Philadelphia, Pennsylvania

I AM always interested in the comments of people when they learn that I teach kindergarten. Immediately they seem to feel that I am "lucky" or relieved of many responsibilities encountered by the grade teachers. The majority of people feel that kindergarten is a place where children cut, sing, paste, and enjoy rhythms until they reach the age of six. Unfortunately, many kindergartens serve for that purpose alone. Ideally, the kindergarten serves a far greater purpose. It is a place where children develop as individuals and prepare for future studies.

Experience demonstrated that trips can be exceedingly meaningful to youngsters as they try to understand better the world in which they live. Trips have helped them to create original stories and songs for our collections. The pupils have often relived their trip experiences in block building, clay modeling, paper work, and during discussion periods. After one of our weekly trips in the neighborhood, the class decided that they would like to have a store in the classroom. We discussed the matter to be sure that it was more than a passing fancy and decided that we would have a grocery store. Immediately each pupil decided and made known what he would like to contribute to the self-appointed committees. Within a relatively short period of time our store was constructed and a plan prepared whereby the job as store clerk would rotate weekly.

When the store began to function smoothly, various questions were asked about prices and making change for the customers. Then I was actually confronted with the problem: "How much arithmetic should be included in the kindergarten curriculum"? I had never given the group any type of formal examinations but I was convinced that they were ready for the very early lessons in arithmetic.

According to Wheat,[1] testing of ability on this age level should be informal and I followed through on this idea. I asked each child to count as far as he could and then listed next to each name the abilities and difficulties manifested. I learned that the youngsters could count objects ranging from ten to one hundred. I realized that the ability to count was not indicative of a readiness for arithmetic for counting might have been learned through a meaningless drill or under the influence of a strong paternal or maternal hand.

When we speak of readiness I am always tempted to ask "Just how do we really know when a child is ready?" I am cognizant of the fact that a child will show a readiness after he has mastered what has gone on before or when he shows an anxiousness for new assignments; but readiness, as Brownell[2] defines it, includes all aspects of ability. He further states that involved in this ability are the dynamic factors of motivations and the social factors of needs. I can go along with Brownell for it has been my observation that children learn to count or build up number concepts as it serves a need and interest. When I was able to make clear to the youngsters the need for counting as the store clerk, as the customers, or just as a person, they realized the importance and were motivated to count. I further explained and showed the class how we used counting for block building, table setting, giving out supplies and various

[1] H. G. Wheat, *The Psychology and Teaching of Arithmetic* (Boston: D. C. Heath and Company, 1937), page 168.

[2] W. A. Brownell, "Readiness and the Arithmetic Curriculum," *Elementary School Journal*, 38 (January 1938), 344.

other activities in the classroom.

Although children might show an outward readiness for arithmetic we must consider the inner man. The article by Dickey[3] provoked much thought and contributed to my thinking out this problem of arithmetic readiness in young children. "Readiness," as defined by Dickey, "is a concept designed to tie together the overt behavior (symptoms) of the learner with the inner dynamics of maturation." He further states that "it (i.e. readiness) is the bridge connecting the observable behavior of the learner with the organized covert behavior operating on his level of maturation." Readiness includes the physical, emotional, mental and social status of the child. It is also important to remember that learning is an orderly process based on maturation.

This entire concern for readiness on the part of the pupil before presenting new facts and ideas is the result of a change in our philosophy of education. There is now a broader emphasis on growth and development of the personality including social-utility objectives.[4] Psychologically we have changed our concept also for now we have the broad organismic concept of the learning and creative processes which emphasizes the roles of maturation. The goals and insights into the problems are always in view.

In view of the fact that the children were eager to use arithmetic and possessed the ability and readiness, I started an orderly systematic study of arithmetic that has led to a clear, exact and definite learning. We counted tongue depressors individually and collectively, picking up a stick with each number. We counted the stairs as we ascended to the roof and descended to our classroom. Each day one child was permitted to count the number of children present and give the number to the porter who in turn would put down the cots for our afternoon rest period. Occasionally I would allow an error to go through and then the experience would be more significant when the person that counted discovered that there were too many cots or not enough cots. When children discover their own errors the correction is more meaningful.

Although I was not firmly convinced that I was actually doing the right thing when I included orderly arithmetic lessons in my curriculum, I felt that it would be more harmful to let the ideas and skills possessed by the pupils in the group lie dormant and unused, or unchallenged.

I have been convinced as I work with five-year-olds that we underestimate the ability of children in the kindergarten to use numbers. It is not necessary to hold back the teaching of arithmetic but it is necessary to bring forth some meaningful and useful teaching of arithmetic in the early grades.

This experience with numbers has not been an unhappy experience for anyone for I stand alert watching for clues of disturbances and repeated failures. Furthermore, the children are not required to count but they are motivated to learn because it will subsequently mean assignment to the tasks involving counting. There are several youngsters in the class who require individual attention and others who cannot count beyond twenty but in neither case is it an indication of failure.

I can recall three definitions I learned some time ago which are significant in such a discussion. 1) Maturation is the growth of a structure as it reacts to various stimulations surrounding it. 2) Learning is growth in response to the functioning of a structure. 3) Development is the perfection of functions. They all remind us of one important factor . . . a child must follow the laws of his own growth although he may not show a readiness equal to other members of his class.

Development was encouraged through many experiences. The store was not the

[3] J. W. Dickey, "Readiness for Arithmetic," *Elementary School Journal*, 40 (April 1940), 593.
[4] *Ibid.*, p. 592.

only activity that involved the use of numbers but also such daily tasks as setting the tables, giving out paper, or scissors, taking attendance, or selecting the four records for use before lunch. Many times the class formed a line and I asked my helper to send four children to the washrooms. As a safety measure we limited the number of children in any particular area according to space. For example: three children could use the work bench together, not more than six could use the dramatic play area, and as many as five children could use the science materials. Many times I found more than the limited amount and asked the children to count and see whether we didn't have too many in the group.

After the pupils learned to count they would apply their knowledge to every conceivable aspect of the classroom activity. One day I saw some of them clustered around our assignment calendar which gives names (and an identifying sticker next to each name) of the persons responsible for watering the plants, feeding the fish, and setting the tables each day. They were busily and accurately counting "how many days before I water the plants."

Many of our games involved the use of arithmetic. I often counted aloud to see how fast they cleaned up after an activity period. Subtraction (a topic initiated by a pupil) was demonstrated as merely a matter of regrouping. I used the game of musical chairs to illustrate the point. We started with all the group on one side of the room. We counted the number and then placed a chair in the center of the room for each child except one. As the music played the group would dance around the chairs but when the music stopped everyone grabbed a chair. In view of the fact that we were short of one chair someone that wasn't quite fast enough didn't get a chair. The child would go to the other side of the room. After I had enough children on both sides of the room I would explain the matter of subtraction as nothing more than regrouping.

It is axiomatic that we can teach a child to count through drill. I have taught successfully to a large number of youngsters such mathematical terms as *half, almost all, large, small, biggest, widest, highest,* etc., but I have not forced the remainder of the class to learn the material. A child will learn when he is ready, interested or feels the need. Above all, a child will learn when he understands.

I can agree with some of the results found in the MacLatchy study of the preschool child as described by Carolyn Stotlar.[5] Children under six can asquire a considerable understanding of numbers and some ability in dealing with number situations. The important factor in all presentations of arithmetic problems is to teach with meaningful concrete objects, especially to young children, and to stress understanding. The abstract number problems should enter the program after a firm foundation has been tried and established. Abstract numbers in the initial arithmetic lessons are void of any understanding.

Number concepts are important and should be developed as early as possible for they provide the ideas necessary to understand facts and processes. In Grades I and II concepts are vital for one aspect leads to accuracy in counting and another shows the relationship between cardinal number names and ordinal ideas. Still another aspect deals with the idea of tens and ones and another shows the relationship between number names, numerals and quantities. We in the kindergarten age group are limited to the number of concepts we can build up but I feel that our discussions of arithmetic and its purposes and uses will be helpful in preparing children for Grade I. At least they won't be stunned by reading and arithmetic simultaneously. It is gratifying to hear a pupil ask for a whole sheet of construction paper "So I can make a big house" or to

[5] Carolyn Stotler, "Arithmetic Concepts of Pre-School Children," *Elementary School Journal*, 46 (February 1946) 342–345.

have a pupil ask for a half of a sheet and then ask whether he might fold it in half.

Recently I looked over at the block-building area and saw Ned standing forlorn, looking at his incomplete building. I went over to find out just what could have happened to cause his helpless appearance. He told me with deep regret that he couldn't finish his building because all of the long blocks were taken and he needed more. Before I could help him solve his problem Bob who usually has little to offer (constructively) shouted, "Use two short blocks like we did and it makes a long block." I didn't teach that fact to Bob nor any other youngster and couldn't accurately tell how he discovered that correct observation but it takes me back to a statement made by Florence Reid,[6] "Rational counting is an outgrowth of the child's development."

Occasionally it is felt that the task of teaching kindergarteners to count and build up number concepts is an easy one. Quite to the contrary; the task is not easy but can be achieved. I have assured the children that academic skills are secondary and have placed emphasis on their security and happiness in the group.

The following is a brief concise record of the progress of one child made over a four month period and will indicate the possibility for development if only the opportunity is given:

February 8—C's first day as store clerk. "Teacher how many pennies should I get for eggs?" (I answered fifty pennies.) C began to count and went to 20 without requiring any assistance. Continued task without trouble for articles less than 20 cents.

February 16—C's turn to set the table. Counted the children one by one and set the table for 18 children. At lunch he discovered that he needed two more settings and went to the kitchen for additional supplies. "Teacher, I made a mistake." I explained to him that a mistake is very possible and he should recount for the correct number. C had quite a bit of difficulty for the children just would not stay together.

February 19—C compared his building with his friend's building and decided that he needed "this many more blocks" holding up seven fingers. C sang a counting song as he added blocks to his building.

February 23—C asked to serve as store clerk because the appointed child was absent. He counted his money and then turned to me. "This is how my grocer counts." He yelled at M, "This store is not open yet—don't you see I'm counting my money?" Later M walked in and gave C what she thought was enough money for her purchase. C took a long time counting each penny as he placed it in the register. Finally, he discovered that he was short and ran out of the store into the doll corner where M was playing the role of mother. "You are a gypper. You didn't give me all of my pennies. Don't you think I can count?" She disagreed and C called me to the scene. We agreed that we couldn't check the amount M had given because other money was in the register at the time. So we suggested that hereafter C should count the money before putting it in the register. C continued as clerk and counted the money slowly before permitting the customers to leave the store.

February 28—"Teacher, there are no more days in February." I asked C how many days there were in February and he counted with unwanted but definite assistance from classmates. "Now we must go to the *next* page of the calendar."

March 1—"Miss Loretta this is a long month." "Why, C?" I asked. "Because there are *more* days than we had last time." C counted 31 days to me without trouble although he slowed up from 29 to 30. C went to tell and prove to the other children that March would be a long month.

March 6—C counted with excitement the number of baby guppies we have. (He now counts rapidly and with a great deal of self confidence. He loves to count and goes around counting every object in the room.) I was particularly interested in his progress because in the beginning he tended to shy away from our very first counting experiences.

March 7—C counted the children for the porter and gave him the correct number. "Be sure you put down 23 cots, Tim. Boy, will we need a lot of room for cots today . . . everybody came to school!" C counted each cot as it was put in place and I was impressed that he was able to tell Tim that he needed three more cots after 20 had been set up.

I don't know whether C's progress is evident to the reader but he has gone a long way to appreciate and use numbers. To him arithmetic is fun and he enjoys counting to see how far he can go. C told me that he wanted to be a number man when he grows up and I chuckled for I thought of the common connotation of the

[6] Florence Reid, "Incidental Number Situations in First Grade," *Journal of Educational Research*, 30: 36.

title. (A number man is one who collects numbers for gambling purposes.) I explained to C the many ways we use numbers and no matter what he really decides to be when he grows up, he will use numbers.

March 19—C returned after a short illness. "Miss Loretta, did I miss any new counting games?" "Yes," I replied, "but we will play them again so you may learn them." "Gee, I'll be so glad." I have often wished that I could have recorded all of C's expressions with all the zeal that he showed. C went over to the crayon can later in the day and counted each piece of crayon and grouped the pieces according to colors. "Look Miss Loretta, we have more yellows than reds. The *next biggest* color is green." C ran out into the hall and came back shouting, "Teacher, when the long hand moves to the next number it will be clean-up time." "Thank you, C." C could not tell time but he knew the position of the hands for clean-up time, lunch time, and snack time.

March 28—We had visitors and C put on a perfect show for them. "Do you know my teacher's name? Her name is Miss Loretta. Do you know what my name is? My name is C. Miss Loretta has a lot of children today and they are so noisy. Do you want me to count for you?" C counted, but I lost track to go to another child's rescue.

April 10—C went over to the calendar and counted how many days before the holiday. His number was correct. "Teacher, did I ever count to 100 for you?" "Why no," I answered, "would you like to right now?" C started to count but I stopped him to get the tongue depressors. He started and counted to 100 taking out a depressor for each number.

In summation, by this time (early May) C can count to 100 and enjoys counting. He wants to do subtraction and addition the way "daddy" does but I assured him that the first-grade teacher will teach him more number work. He is anxious and ready for new work and I feel he has benefited from his early experiences in the kindergarten group. He learned the early elements of number work in a free relaxed group and it made quite a difference.

I wasn't sure that kindergarten was the right place to begin getting children arithmetic conscious, but I am convinced that there is a place for arithmetic in kindergarten if it is taught with meaning. The important factor to remember is that the children must show a readiness for the subject matter and must know the social value of the subject matter.

ANNOUNCEMENT OF 1951 NCTM CHRISTMAS MEETING

The 1951 Christmas meeting of the National Council of Teachers of Mathematics will be held at the Oklahoma A. & M. College, Stillwater, Oklahoma, December 27, 28, and 29. Plan to attend and profit by the excellent program prepared by Miss Lenore John, vice-president of the Council. The complete program may be found in the October issue of THE MATHEMATICS TEACHER.

Travel Information

Those traveling by train should arrive in either Perry, Tulsa or Oklahoma City, Oklahoma and proceed to Stillwater by bus. Since adequate bus service is not available in Perry for all trains, transportation will be provided if the Department of Mathematics at the College is notified by December 15. Those traveling by plane will probably have to come from Tulsa or Oklahoma City by bus, though it is possible that air service may be available by the time of the meeting. Those traveling by car will find excellent highways leading into Stillwater from all directions.

Sante Fe trains arrive at Perry at the following times: from the north at 7:38 A.M., 3:50 P.M. and 11:22 P.M.; from the south at 6:05 A.M., 7:50 A.M., 2:20 P.M. and 7:05 P.M.

Frisco trains arrive in Tulsa at the following times: from the north at 5:30 A.M., 7:35 A.M., 10:00 A.M., and 2:15 P.M.; from the south 7:40 P.M. and 9:25 P.M.

Self-Directed Homework

By Rose Klein
Somers Junior High School, Brooklyn, N. Y.

All of us know classes in which, the lesson over, the teacher calls out, "Just a minute, class. Homework for tomorrow: page 00, ex. 1–5." Often it is not as rushed as this. Quite calmly, about the middle of the lesson, the teacher announces the stereotyped words, "And now the homework: page 00, ex. 1–5."

What are some objections to the above?

(1) Must *all* pupils have the *same* assignment? Definitely not. If pupil A can do a new type problem expertly at the end of the lesson, and pupil B is still slow at this new work, why should both be given identical assignments? It is definitely unfair to pupil A.

(2) The *teacher* has decided what tasks are to be done. The pupil has no choice in the matter. Let me explain later why this is so unwise a procedure.

(3) The teacher has to inspect, or supervise the inspection of about 40 homework papers in each class. What a waste of valuable time!

(4) The old-type homework assignment leaves many pupils tense and under pressure. Lines must be ruled just so, the papers must be exceptionally neat, etc. Copying is widespread. A conscientious pupil forgoes many opportunities to add to his cultural background, or forgoes opportunities for social contacts which are so important during the teen age. Considering school, homework, music lessons, dancing lessons, dramatic lessons, religious studies, there doesn't seem too much time left for relaxation, for free play.

Here, in the matter of homework assignments, is a wonderful opportunity for pupils to get training in assuming responsibility. We're not dealing with infants to whom mothers continually say, "Do this," "Do that." No wonder some children rebel. Mothers order them around, teachers order them around in school, and even after school indirectly by the usual homework. Under the old method the pupils do what they are *ordered* to do, not what they would *like* to do. It's *work*, not *play*. I propose to make it *play*. Let the pupils plan so that there will be fun in *what they* are going to do, and *how they* are going to do it, and let them give a fairly good reason *why they* are going to do this. A group of girls and boys in one class became so interested in this newer type of work, they met at least once a week in the community center of our school to discuss mathematical topics. It was fun to them, holding initiations suitable for a mathematics club, and doing everything by themselves.

How can we make homework a game? Pupils are filled with wonder, and *curiosity*. They can ask endless questions to which many a parent will testify. Let's use this fact in our homework assignments. Pupils have *hobbies*. Let's use this fact too. Pupils have varied interests, whe her because of background, or future goals. Let's use the idea of *varied interests*.

As an idea pops into some child's head, he may ask a definite question, "How? Why? When did—? Could we do this—?." Make that the basis for the homework assignment. Tell the pupil to look it up and report to the class, or appoint him chairman of a committee to investigate this matter and then to report to the class. In that way, almost everyone has an individual assignment. Perhaps you may even want to challenge the class, "See who will be the first to get Johnny an answer. I wonder if someone will get him an answer by tomorrow."

If a pupil is unimaginative, or has not been sufficiently stimulated to seek knowledge, he doesn't do project work for the time being. Perhaps at the moment, he has too many other things on his mind (Confirmation, music recital, etc.). Leave him alone. Sooner or later, he will be aroused and eager to look up something. However, this does not mean that the person has no homework. There is another part to the daily assignment.

Each pupil must assume responsibility for mastering the day's lesson and for

reviewing previous lessons. All work done in class is kept in a notebook. Everything in this notebook is corrected before the class is dismissed. Therefore, when a pupil wishes to review the work, he has model examples with which he can practise. If he hides the work, just exposing "Given" and "To Find," or the introduction, he can give himself a practise test on either new or old work. If I have reason to believe that a pupil who needs this review is not doing it, I ask him to hand in his practise papers each day until further notice. Others may quickly glance through the notebook and call that a review. His must be in writing, on paper to hand in. If the pupil needs practise beyond what is provided by the classwork, he resorts to the text, choosing his own page, and doing as many examples as he wishes. Upon showing considerable sustained improvement, the pupil is then told he doesn't have to hand in his work in the future.

My objection to ex. 1–5 was that this was *habitually superimposed* by the teacher on an *entire* class. The newer plan provides for a flexible type of homework which can be adjusted to meet the needs of the individual. Some individuals need more than 5 examples, some fewer. Notice that the pupil chooses which type of example he needs to practise, whether this be new or old work, or both. No two homework papers are alike. Any example in the notebook, or later in the textbook, may be chosen, and *any number of these*. How grown-up the pupil feels. He is the boss, choosing his own homework, stopping when he feels like stopping. He decides when the day's practise should be ended. Look at the way his personality expands as he is given free rein and is allowed to *make decisions for himself*. If we consider the pupil's development at this adolescent stage, we note the parent is dropping the reins by giving the child an allowance, letting *him decide* how it is to be spent, and also letting *him decide* on his own wardrobe. Teachers should drop the reins in this matter of *what* homework and *how much*, for the adolescent *is* mature enough to decide these for himself.

You may find a pupil handing in work on topics he knows. This is quite rare, and can be settled by merely pointing out to him that those aren't the topics he is poor in. At the end of every test, and often during classwork, entries are made on a page in the notebook called "Topics I Forgot." This is compulsory after a test. From this list the pupil chooses the topics for his reviews.

Very often I do give a definite assignment, especially on fundamentals, but usually to only a few *individuals*. One pupil may be given drill on division with fractions, another, another page—division with decimals, another may get more problems, say, in commercial discount. You will observe, however, that I *do not* have 40 papers to inspect in *each* class.

Notice, there is intelligent action and purposeful activity *by* the pupils. However, there is also intelligent direction and motivation by the teacher. If a pupil is at all doubtful of his ability to do the grade work, he practises. If you are doubtful of his ability, you ask that this review be in writing, and handed in *every* day (excluding Monday) until further notice. The children do not have complete freedom regardless of the consequences. The pupil shoulders the responsibility, but this is contingent upon the pupil's maintaining work compatible with his ability. The teacher is still in control, though unobtrusively.

Let us go back to the first part of the assignment, which is the purely voluntary work, wherein pupils answer the Hows, Whys, Whens, What ifs—research projects. When a pupil brings in a report, it must be in his own words, or in language on a junior high school level. There must be understanding on the part of the pupil. No one gains from a parrot-like repetition of an article in the encyclopedia or other source. When a pupil brings in a project, for example a sundial he has constructed, this is not just a copy of the model pic-

tured in the book. The pupil must be able to answer in his own words, the question "How does it work?" and be able to convey his knowledge to other members of the class. Another word of caution. These reports should not be the result of aimless browsing, even though browsing has great value. The pupil just doesn't choose any mathematical topic in the encyclopedia. There should be *directed* search arising out of some classroom situation. To illustrate. As a rule, a pupil in a seventh year class would have no occasion to make a report on logarithms, assuming the pupil understands the idea. This would be foreign matter for the class, uncalled for, and totally out of place. On the other hand, a pupil could give a report on cantilever and suspension bridges when the class is studying about triangles. That is apropos, and quite fascinating to the pupils. The pupil who is very much interested volunteers to give such a report because the topic of interesting bridges was mentioned in class, or pertains to the class work.

In conclusion, if a fellow brings in mathematical material which he obtained from his uncle's office, the pupil will peruse that, and learn more about mathematics in real life, than if you said "Homework, ex. 1–5." Pupils have brought in salesman's rate books from insurance companies, income tax charts, sample copies of civil service tests, slide rules, sundials, architect's blueprints of an entire two-story house, booklets on actuarial work and on navigation, even library books on mathematics (Careers for Girls in Science and Engineering). One girl started a scrap book of candy wrappers and labels from canned goods, to show that many commercial companies in America are using the metric system. You'd be surprised at the store of knowledge, and the sources.

Let's give pupils more training in leadership. Give the pupils the responsibility and practise in making decisions.

Earlier in this report I mentioned a group deciding on an initiation suitable for a mathematics club. This is what one boy suggested, and was enthusiastically accepted by the group, that "all members had to wear around their necks a paper chain made from paper about 3 inches square." This, some of you will recognize, is the old problem of taking a small paper, making a small hole in it, and challenging others to make the hole large enough for you to put your foot through. The pupil changed it to, large enough to put your head through.

What are the pupil's reactions to self-directed homework?

Slow pupils:

(1) "I like it (individualized homework) because it is going to help *me*."
(2) "That's a topic I never knew about until now."
(3) "I learned three topics, so far this week. I've really improved a lot."

Normal:

"When I need written work, I do it. Can't hurt me, but can help me." (Like vitamins.)

And the bright pupils:

(1) "We get enough practise in class."
(2) "This way I have more time to learn from people, not just from books. When I go down to my uncle's place, I see where mathematics is used in real life."
(3) "Now I can call some time my own."

As for the teacher:

(1) She doesn't have to answer the question, "Do we have to have homework?" Each pupil concerned knows why.
(2) No text has as yet been written which is challenging enough to bright pupils.
(3) The teacher does not have 40 homework papers from each class. She therefore can devote more time to *planning* work to meet the needs of individuals.
(4) Mathematics is not a series of tricks. Pupils gain the power to organize knowledge (reflective thinking).

What does the guidance counselor have to say? She agrees that

(1) This method keeps the child continually examining his own progress and playfully intrigues the child into further effort.
(2) The teacher knows her pupils and their ability.
(3) Pupils are helped to cross the Rubicon from childhood over adolescence into adult ways.

Anyone, teacher or parent, should welcome an opportune moment for adolescents to make independent decisions.

Critical Thinking—What Is It?*

By Robert E. Pingry
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University of Illinois, Urbana, Illinois

What is *critical thinking?* Can we mathematics teachers tell what we mean by the phrase? Do mathematics teachers mean the same by critical thinking as do the teachers of the social studies or the logician or the psychologist?

Kenneth Brown recorded the replies of five hundred classroom teachers concerning the important objectives of the teaching of geometry.[1] The objective receiving the highest rating was, "To develop the habit of clear thinking and precise expression." This is probably a noble objective, but what does it mean? What is the habit of clear thinking? In fairness to Kenneth Brown and the five hundred teachers, it should be stated that the wording of this objective was not theirs. The objective had already been printed on a questionnaire first distributed by Mr. Shibli in 1930 and more recently by Kenneth Brown. It is true, however, that in recent mathematics meetings and in recent articles, phrases concerning *clear thinking* have been used in just as vague a way as the objective printed on this questionnaire.

The title of this paper concerns itself with *critical thinking*. How is *critical* thinking different from *clear* thinking, or is it? If this troubles you, then pity the poor reader who is faced with an avalanche of descriptive phrases of thinking as he reads through the journals and books on teaching, philosophy, and psychology.

The writer has found the following phrases in his reading:

clear thinking
straight thinking
reflective thinking
elaborative thinking
scientific thinking
postulational thinking
autonomous thinking
organic thinking
human thinking
subhuman thinking
logical thinking
rational thinking
factual thinking
inventive, productive, creative, empirical, hypological, and on and on and on . . .

When we mathematics teachers state that one of the objectives of a course in mathematics is to develop the habit of clear thinking and precise expression, aren't we guilty of not being very clear and precise in our own thinking? Possibly J. W. Young was wise when he avoided all of the unusual adjectives to describe thinking and said, " . . . it is clear that the chief end of mathematical study must be to make the pupil think."[2]

The Many Aspects of Critical Thinking

It would be an interesting experiment to ask a group of persons to draw up a list of behavior patterns of people who had learned to think critically. Certainly many differences in the concepts of critical thinking would be apparent. In reading on this subject the writer has been aware of at least five different emphases that have been placed upon the concept of critical thinking. They are:

1. Critical thinking as collecting data,

* A summary of a paper presented to the Mathematics Section of the Illinois Education Association, Lake Shore Division, October 20 and 23, 1950.

[1] Kenneth E. Brown, "Why Teach Geometry?," The Mathematics Teacher, March, 1950, p. 104.

[2] J. W. Young, *Fundamental Concepts of Algebra and Geometry* (New York: The Macmillan Co., 1936), p. 4.

organizing data, and formulating hypotheses from data.
2. Critical thinking as use of correct principles of logic and understanding the nature of proof.
3. Critical thinking as criticism of thinking.
4. Critical thinking as related to understanding of the psychology of propaganda and advertising techniques.
5. Critical thinking as synonymous with problem solving.

Critical Thinking as Skill in Collecting and Analyzing Data

Some would state that they would consider a student a good critical thinker who was good at collecting and organizing data related to a problem; who could evaluate the appropriateness and the validity of sources and information derived from them; who could distinguish between fact and opinion; who could compare data making use of graphs, tables, and statistics; and who had developed the habit of suspending judgment until sufficient data was present. All of these skills and habits have been used by various people as a basis for measurement of a student's ability to do critical thinking.[3,4]

Critical Thinking as Understanding and Skill in the Use of the Principles of Logic and the Nature of Proof

Others would consider those behavior patterns related to the students' understanding of the principles of logical reasoning and the nature of proof as the behavior patterns desired in our students.[5,6] Such outcomes of learning, among others, would be:

1. The ability to recognize the basic assumptions upon which a conclusion depends.
2. The ability to recognize that certain tacit assumptions have been made and ability to identify the tacit assumptions.
3. The ability to detect *reasoning in a circle* in an argument.
4. The ability to detect *non-sequitur* situations in an argument.
5. The understanding that proof by elimination depends upon knowing that at least one of the list of possibilities must be true.
6. The understanding that crucial words and phrases must be precisely defined.
7. The understanding that if one accepts certain assumptions one must accept the conclusions that are logical consequences of these assumptions.
8. The understanding that some people have an inclination to accept a converse as true just because the original proposition is known to be true.
9. The understanding that a converse need not be true just because the original proposition was true.
10. The understanding that no demonstration really proves anything that is not already implied in the assumptions.
11. The understanding that in a deductive system some words must be left undefined.
12. The understanding of the difference between true conclusions and valid conclusions.

[3] M. F. Rosskopf, "The Present State of Evaluation of Critical Thinking in Algebra and Geometry," THE MATHEMATICS TEACHER, April, 1950, pp. 143–148.

[4] Hilda Taba, "The Evaluation of Critical Thinking," *Teaching Critical Thinking in the Social Studies*, Part Four, Thirteenth Yearbook, National Council for the Social Studies, 1942, pp. 123–175.

[5] Progressive Education Association, *Appraising and Recording Student Progress* (New York: Harper and Brothers, 1942), Chapter II, "Aspects of Thinking."

[6] Harold P. Fawcett, *The Nature of Proof*, Thirteenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications, Teachers College, Columbia University, 1938).

Critical Thinking as Criticism of Thinking

Max Black uses the adjective *critical* in connection with thinking to mean *critical of thinking*. He defines logic as the art and science of criticism of reasoning.[7] He further states:[8]

> The logician searches for *standards* of reasoning. To be in a position to *improve* reasoning means to be in a position to distinguish *good* reasoning from *bad*. A man who judges cattle has some specifications before him (often in a very precise form) of what constitutes a good specimen of the breed he is judging; a thinker who tries to improve his thinking must, likewise, have in mind some standard for discriminating better thinking from worse.

Critical Thinking Related to Understanding of the Psychology of Propaganda and Advertising Techniques

Others would include behavior patterns associated with the students' understanding of the psychological basis for propaganda and advertising techniques. They believe it is not enough for the student to recognize the bad reasoning involved in propaganda and advertising, but the student should also understand some of the psychological bases of why the techniques work so well.[9]

Critical Thinking as Synonymous with Problem Solving

Confusion as to the meaning of the term *critical thinking* also arises when some consider a person good at critical thinking who is able to solve problems. Critical thinking is thought of as a synonym for problem-solving or productivity in thinking. A person who is able to attack an original problem situation and quickly obtain a solution is said to be a good thinker. The person who is able to forge ahead into uncharted areas, make generalizations, be creative, make discoveries, and be inventive is thought by some to be a good thinker. Is productivity in thinking the same as critical thinking?

Wertheimer's very interesting and valuable book has made an important contribution in this area.[10] Wertheimer is skeptical of the traditional logic approach to what he calls productive thinking. He writes:[11]

> If ones tries to describe processes of genuine thinking in terms of formal traditional logic, the results are often unsatisfactory; one has, then, a series of correct operations, but the sense of the process and what was vital, forceful, creative in it seems somehow to have evaporated in the formulations. . . . Training in traditional logic is not to be disparaged: it leads to stringency and rigor in each step, it contributes to critical-mindedness, but it does not, in itself, seem to give rise to productive thinking. In short, there is the danger of being empty and senseless, though exact; and there is always the difficulty with regard to real productiveness.

THE COMPLETE THOUGHT PROCESS

One of the most helpful analyses of the complete thought process is that provided by John Dewey in his book, *How We Think*.[12] He analyzed what he called reflective thinking or problem-solving into the following five steps:

1. Becoming aware of a problem.
   A perplexing situation has arisen. The individual is aware that a problem is present.
2. Defining the problem.
   The rather vague, perplexing situation is now made more sharp and specific. The goal to be achieved is now known.
3. Suggesting hypotheses for the solution of the problem.
   Conjectures, suppositions, guesses are made. "The inference involves a leap, a jump, the propriety of which cannot be absolutely warranted in advance."
4. Reasoning out implications of the suggested hypotheses.

[7] Max Black, *Critical Thinking* (New York: Prentice-Hall, 1946), p. 7.

[8] *Ibid.*

[9] C. R. Miller, *How to Detect and Analyze Propaganda* (New York: Town Hall, 1939).

[10] Max Wertheimer, *Productive Thinking* (New York: Harper and Brothers, 1945).

[11] *Ibid.*, pp. 10–11.

[12] John Dewey, *How We Think* (Boston: D. C. Heath and Company, 1933), pp. 106–115.

Tracing out the consequences of the various hypotheses.

5. Experimental corroboration.
   A test of the hypothesis is made against experience.

Although it is realized that the thought process probably does not follow such a clearly defined sequence of steps, it is believed that the analysis can serve as a very helpful outline. The problem solver probably jumps around among the steps considerably before he is able to solve the problem, but as a general outline to help us understand thinking the analysis is very useful. The many behavior characteristics of students doing critical thinking listed in this paper could be classified under the five steps of the analysis. For example, those behavior characteristics that were listed under the headings *principles of logic* and *the nature of proof* could be classified under point four of Dewey's analysis. Wertheimer's emphasis upon *productive thinking* might be classified under point three—that dealing with the forming of hypotheses.

The Mathematics Teacher's Job

As mathematics teachers, our first responsibility is to the student. We should help him grow and develop in all areas. We are responsible for helping him grow and develop in his ability in the entire area of problem-solving as outlined by Dewey. As a part of this total responsibility we must do a good job of helping some of them gain understandings and skills in mathematics. Because of the great number of attitudes, understandings, and skills associated with the process of critical thinking, the problem of training for critical thinking is too great for the mathematics teacher alone. Especially is this true if mathematics teachers limit their efforts to the tenth year geometry course. We mathematics teachers also have some mathematics to teach—for the students' own good. The problem of critical thinking is the problem of the entire school.

It is believed, however, that because of the nature of mathematics, the mathematics teacher is in a position to help the student in many of the aspects of the complete thought process as outlined by Dewey. The Fifteenth Yearbook of the National Council has provided us with some help in this respect.[13]

Robert Thorndike, for a recent publication,[14] wrote on the subject, "How Children Learn the Principles and Techniques of Problem Solving." He listed some specific suggestions as to what schools and teachers could do to foster the problem-solving abilities of students. Many of his suggestions, although made for teachers of any subject, can be of considerable help to us as mathematics teachers.

Thorndike uses Dewey's outline to group his suggestions. He considers point one, becoming aware of problems, and lists several suggestions under that aspect. He does the same for points two, three, and four.

One of the suggestions Thorndike emphasized was to help the student build up a rich background of experiences. He pointed out that the wider the background of knowledge, skills, and understandings of the student the better are his chances for success in problem-solving. It is especially important to note that the knowledge, skills, and understandings must be built up in the area in which the thinking is to be done. If the student is to think about quantitative problems then he must know some mathematics. If he is to do critical thinking about social problems then he must have a background of knowledge in social problems.

Thorndike also pointed out the impor-

[13] National Council of Teachers of Mathematics, Fifteenth Yearbook, *The Place of Mathematics in Secondary Education* (New York: Bureau of Publications, Teachers College, Columbia University, 1940), pp. 37–41.

[14] Robert L. Thorndike, "How Children Learn the Principles and Techniques of Problem Solving," *Learning and Instruction*, Forty-Ninth Yearbook, Part I, National Society for the Study of Education (Chicago: The University of Chicago Press, 1950), pp. 192–216.

tance of certain attitudes in relation to problem-solving. One important attitude is that of suspending judgment until sufficient evidence is available and present. The mathematics teacher can contribute to the building of this attitude. Abundant opportunities arise in the usual mathematics class for pointing out to students or letting them discover that they have jumped to conclusions too quickly.

In relation to Dewey's fourth step, that of reasoning out the implications of suggested hypotheses, Thorndike stated that the skills required are those of "if-then" thinking, of logical reasoning.[15] Mathematics teachers have a very fine opportunity to help the students with the required skills of critical thinking if they teach the subject in such a way as to emphasize the principles of logic and "if-then" thinking.

The Harvard Committee Report states:[16]

> Mathematics is by no means the only road to an appreciation of abstraction and logical structure. But tactically these ends may be approached most readily through mathematics, particularly with the young. No better example of an abstract logical system for use with adolescents than demonstrative geometry has yet been discovered.

The Fifteenth Yearbook states:[17]

> Especially at the high school level, principles of deductive thinking can most effectively be taught in connection with a well-organized substantial subject such as geometry, which being logical itself and free from personal prejudice, can serve as a yardstick.

[15] *Ibid.*, p. 214.

[16] Harvard Committee Report, *General Education in a Free Society* (Cambridge: Harvard University Press, 1946), p. 162.

[17] National Council of Teachers of Mathematics, *op. cit.*, p. 40.

Summary and Conclusion

1. Critical thinking has a great number of aspects and means many different things to different people. Hence, as a descriptive phrase, *critical thinking* is of little use by itself to describe outcomes of learning. It is necessary and important that the phrase *critical thinking* be defined or supplemented by specific outcomes of learning in terms of actual behavior characteristics and skills desired.

2. High school mathematics teachers can help students develop in many of the aspects of critical thinking. Specifically the mathematics teacher can:

a. help the students have a rich background of experiences in mathematics and in mathematics as related to other fields and to life needs;

b. invent situations that will expose the student to question-provoking experiences;

c. help the students develop attitudes that will lead to thoroughness of study, organization of materials, and suspension of judgment until sufficient data is present;

d. set a climate in the classroom sympathetic to questions and questioning;

e. help the students gain ability in "if-then" thinking and knowledge of some principles of logic; and

f. help the students build a set of standards or norms of good reasoning against which they can compare their own reasoning or the reasoning of others. The mathematics teacher can help the students realize that the method of postulational thinking is not dependent upon the subject matter of mathematics but is applicable to reasoning in non-mathematical areas as well.

Change in Price of Single Copies of The Mathematics Teacher

By action of the Board of Directors of the National Council of Teachers of Mathematics, the cost of all single copies, current number or any back number, of The Mathematics Teacher will be 50¢ per copy. This change will go into effect on January 1, 1952.

Motivation in Mathematics: Its Theoretical Basis, Measurement, and Relationships with Other Factors[1]

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Background and Purposes of the Investigation

A short time ago an instructor was discussing a topic relating to motivation with a class of prospective teachers. To illustrate the point that many motives are learned, a statement resembling the following was made: "Consider motives related to the study of mathematics as an example. Many people develop intense motives to avoid mathematics. Now *there is nothing about mathematics itself that could cause people to dislike it;* this could occur only as a result of unfortunate experiences with it." As the words in italics were uttered, a chorus of chuckles arose from the class. It was apparent from the uniformity of the reaction, and from the discussion that followed, that this idea was totally strange to the thinking of many of the students. A considerable number of them had acquired unfavorable attitudes toward mathematics which were so firmly established that the idea that the study of mathematics could have been enjoyable to them appeared ludicrous. This is especially significant in view of the fact that the class was composed of a select group of college juniors whose classroom experiences with mathematics were probably more pleasant than for most of their classmates in elementary school and high school.

This incident, unfortunately, illustrates a relatively typical rather than a rare attitude. Probably every high school teacher has encountered students who "just hate mathematics." Relatively few students learn to really enjoy mathematical study and to fully appreciate its practical and cultural values.

What are the causes of the development of dislike for mathematics? How can the development of favorable patterns of motives with regard to mathematics be fostered in students? It was toward providing answers to questions such as these that this inquiry was directed.

The primary purposes of the investigation were to draw together, from the literature, the elements of a theory which appears to hold the greatest promise for explaining how motivational patterns develop and to secure evidence which is pertinent to verification of this theory. There were two secondary purposes of the study: (1) to construct a test instrument for measuring certain aspects of motivation with regard to mathematics for use in connection with the primary purposes of the study, and which can be used in other research and in teaching, and (2) to determine the extent to which findings concerning relationships between motivation with regard to mathematics and other factors can be utilized for estimating the motivational status of an individual concerning mathematics.

A Sketch of the Theory and Statement of Hypotheses

The theory of the investigation is closely related to the reward-reinforcement theory of learning. The basis for this approach is that motivation with regard to mathematics is considered an attribute of personality and that most attributes of personality are learned. According to this

[1] This is an abstract of a dissertation completed at The University of Chicago in March, 1951. Members of the faculty advisory committee were M. L. Hartung, R. W. Tyler, and B.S. Bloom.

theory, the basis for learning is the experiencing of pleasure or satisfaction. That is, in the learning of behavior patterns, those responses are learned which repeatedly occur close in point of time to the experiencing of pleasure or satisfaction. Since this reinforcement through pleasure results in a tendency for the behavior pattern to recur, it follows that a motivational state which initiates and directs this behavior has been established. When the individual meets with frustration instead of satisfaction, aggression often follows. One form that this aggression can take, in situations involving mathematics, is to dislike mathematics.

Most pleasures are experienced through satisfaction of the needs of the individual. Likewise, frustration can be defined as the experiencing of blocks to reaching goals which are set by the needs of the individual. The need concept, therefore, is fundamental to the theory of the investigation. Need is defined as a deficiency or want in the organism which causes tension or striving in the organism. Many needs are products of learning and are therefore partly determined by the environment. Need patterns vary from individual to individual, but it appears that there are a few basic needs which are common to essentially all human beings.

The crux of the theory is represented by the following statement of the major hypothesis: *The motivation of an individual with regard to mathematics is affected positively by experiencing pleasure or satisfaction, and negatively by experiencing unpleasantness or frustration, as an outcome of interaction with a situation having mathematical aspects, the extent of the effect on motivation with regard to mathematics being dependent upon the intensity of the satisfaction or frustration and upon the extent to which mathematics is involved in the interaction.*

The major hypothesis was not tested directly in the investigation. Instead, subhypotheses were deduced from it that were more suitable for testing. The first of these sub-hypotheses is the following: *Any factor which has the effect of increasing the amount of satisfaction, or reducing the amount of frustration, in interaction in a situation involving mathematics, has a positive effect on the motivation of the individual with regard to mathematics, and any factor which has the effect of increasing frustration, or decreasing satisfaction, in interaction in a situation involving mathematics, has a negative effect on the motivation of the individual with regard to mathematics.* From this the following sub-hypotheses were deduced:

1. There is a positive correlation between motivation with regard to mathematics and (a) general mental ability, (b) algebra aptitude, (c) geometry aptitude, and (d) achievement in terms of mathematical processes and mathematical understandings.
2. There is a negative correlation between motivation with regard to mathematics and achievement in subjects not related to mathematics.

This same satisfaction-frustration theory leads to the expectation that there are relationships between motivation with regard to mathematics and other aspects of personality, as measured by a personality test—since basic needs of the individual are controlling factors with regard to many aspects of personality, including motivation with regard to mathematics. It seemed inadvisable, however, to formulate specific hypotheses in this regard, because of limitations of the types of personality tests which could be used in this situation. This aspect of the investigation was, therefore, of a broad, exploratory nature.

The Instrument for Measuring Motivation in Mathematics

The first step in drawing up the instrument was to make an analysis of motivation with regard to mathematics to provide a basis for item construction. This analysis was in terms of content, situation, and

behavior—that is, in terms of the kinds of behavior with regard to the various types of mathematical content in various kinds of situations that are indicative of favorable or unfavorable motivation with regard to mathematics.

The items are all of the stereotyped multiple-choice type in which a situation is described or illustrated and several questions asked concerning what the student does, thinks, or feels in such situations. There are three forms of these stereotyped response choices represented by the following sample items:

Type 1. You probably have solved many problems like the following:

James, Bob, and Jerry wished to save money from their jobs to buy a bicycle costing $37.50. James could save $1.25 per week, Bob could save $2.20 per week, and Jerry could save $1.75 per week. How long would they have to save their money at these rates before they could combine their savings and buy the bicycle?

When you work at problems of this kind,

2. How often do you find it difficult to keep your attention from wandering off to other things?

The student was asked to record his responses to Type 1 items on separate answer sheets according to the following key:

A—Always, or at least 9 times out of 10.
B—About three fourths of the time.
C—About one half of the time.
D—About one fourth of the time.
E—Never, or not more than 1 time out of 10.

Type 2. The student was given a list of topics such as

109. How mathematics is used in air navigation.

He was then asked to record his answers as follows:

A—*Definitely would* like to know more about this topic.
B—Uncertain, but might be *a little interested* in knowing more about this topic.
C—Uncertain, but it is *not likely* that you would be interested in knowing more about this topic.
D—*Definitely would NOT* like to know more about this topic.

Type 3. Imagine yourself in the following situation: One of your classmates is telling you that he (or she) has been working on an idea for a way to extract square root which is better than the one commonly used. He (or she) wants to explain it to you, to have you check it over to see if the reasoning is correct, and to try it out to see how it works.

25. Would you be interested in hearing the idea explained?

Responses to items of this type were recorded according to the following key:

A—YES
B—Uncertain, but probably YES
C—Uncertain, but probably NO
D—NO

There are 142 items of Type 1, 134 items of Type 2, and 43 items of Type 3. It is hoped that further studies of the instrument itself will make possible a reduction in the number of items without decreasing its usefulness.

The instrument was designed so that it could be used with IBM electrically scored answer sheets. It can be scored in terms of a single index of the student's general motivation with regard to mathematics, or in terms of various combinations of part scores. A student's most intense interests and areas of most favorable or unfavorable motivation can be determined by inspection by superimposing specially cut templates over the answer sheets.

Three approaches were used in determining the validity of the instrument:

1. The case for face validity was presented.

2. The relationships between scores on the instrument and other facts about the student which are indicative of motivation with regard to mathematics were studied:

a. Teacher ratings of the motivation with regard to mathematics of the students.

b. Estimates based on rankings by students of subjects liked best.

c. Participation or non-participation in voluntary mathematical activities.

3. The results of testing the subhypotheses were examined in terms of implications for the validity of the instrument.

The results of these studies of validity

are generally favorable to retention of the hypothesis that the instrument is valid, but more conclusive evidence is desirable.

An estimate of the reliability of the instrument was obtained by the split-half method. The correlation between the two halves was found to be .945, using 153 cases. Application of the Spearman-Brown correction formula raised the estimate to .972.

Collection and Analysis of Data

Data for testing the sub-hypotheses were secured by administering tests and questionnaire forms on the factors involved to the ninth grade students in a high school in a suburb of a large mid-western city. Additional data were secured from the school records, scores on tests administered as a part of the regular testing program of the school, and teacher ratings. The types of data of greatest importance to the study are listed in Table 1.

The data were analyzed by computing the product-moment correlation coefficients between the scores on the *Motivation Inventory for Mathematics* and measures of other factors. These correlations are shown in Table 1. For each entry in the table a positive correlation coefficient means that an increase in favorable motivation for mathematics is associated with an increase in ability or adjustment, as the case may be. Several partial correlation coefficients were computed also, so that the relationships between motivation with regard to mathematics and other factors, with mental ability held constant, could be determined. These partial correlation coefficients are shown in Table 2.

Since most of the product-moment correlation coefficients were computed with IBM equipment, with which it is about as easy to compute all of the intercorrelations as it is a few, many intercorrelations not directly related to the purposes of the investigation were obtained. Of particular

Table 1

Product-Moment Correlations Between Scores on the Motivation Inventory for Mathematics and Measures of Other Factors

Measures Correlated with Scores on the Motivation Inventory for Mathematics	Correlation Coefficient	Number of Cases
Scores on *Otis Quick-Scoring Test of Mental Ability*	.281	116
Scores on *Iowa Algebra Aptitude Test*	.341	116
Scores on *Iowa Plane Geometry Aptitude Test*	.287	83
Scores on *Stanford Achievement Test* (Arith.)	.261	85
Scores on *A Test of Basic Mathematical Understandings*, by V. J. Glennon	.514	116
Term grades in Algebra	.252	93
Term grades in General Mathematics	.097	83
Term grades in English	.303	176
Term grades in Social Studies	.316	177
Term grades in Latin	.290	54
Term grades in Home Economics	.354	27
Scores on the *California Test of Personality*		
Self Reliance	.294	116
Personal Worth	.115	116
Personal Freedom	.230	116
Feeling of Belonging	.059	116
Withdrawing Tendencies	.081	116
Nervous Symptoms	.117	116
Total Self-Adjustment	.211	116
Social Standards	.166	116
Social Skills	.150	116
Anti-Social Tendencies	.217	116
Family Relations	.071	116
School Relations	.169	116
Community Relations	.095	116
Total Social Adjustment	.221	116
Total Adjustment	.233	116

TABLE 2

PARTIAL CORRELATION COEFFICIENTS BETWEEN MOTIVATION WITH REGARD TO MATHEMATICS AND OTHER FACTORS, WITH MENTAL ABILITY HELD CONSTANT

Measures Correlated with Motivation with Regard to Mathematics	Coefficient of Partial Correlation
Scores on *A Test of Basic Mathematical Understanding*	.455
Scores on the *Stanford Achievement Test* (Arithmetic)	.101
Term grades in Algebra	.122
Term grades in General Mathematics	−.065
Term grades in English	.166
Term grades in Social Studies	.200
Term grades in Latin	.215
Term grades in Home Economics	.255
Self Reliance score of the *California Test of Personality*	.262
Total Adjustment score of the *California Test of Personality*	.184

interest are some of those showing the relationships between the scores on *A Test of Basic Mathematical Understandings* and the other types of data collected for the investigation. These are shown in Table 3. Reference will also be made to the correlation of .573 between algebra aptitude and term grades in algebra. Two additional findings not shown in these tables follow. The partial correlation between scores in basic mathematical understandings and term grades in algebra, with mental ability held constant, was found to be .540. Similarly, the correlation between basic mathematical understandings and term grades in general mathematics, with mental ability held constant, was found to be .252.

SUMMARY OF FINDINGS

The correlations in Table 1 of .281, .341, and .287 between scores on the *Motivation Inventory for Mathematics* and scores on the *Otis Test of Mental Ability*, the *Iowa Algebra Aptitude Test*, and the *Iowa Plane Geometry Aptitude Test*, respectively, provide evidence in agreement with sub-hypotheses 1 (a), 1 (b), and 1 (c): that there is a positive correlation between motivation with regard to mathematics and measures of general mental ability, algebra aptitude, and geometry aptitude. Each of these correlations is significantly positive at the one per cent level of confidence.

Questions arise, however, concerning sub-hypothesis 1 (d): that there is a positive relationship between achievement in mathematics and motivation with regard to mathematics. The correlation of .514 between scores on the *Motivation Inventory for Mathematics* and Glennon's *Test of Basic Mathematical Understandings* provides evidence clearly favorable to this sub-hypothesis in terms of this kind of achievement. However, the correlations between motivation with regard to mathematics of the type measured by the *Moti-*

TABLE 3

Product-Moment Correlations Between Scores in Basic Mathematical Understandings and Measures of Other Factors

Measures Correlated with Scores on Basic Mathematical Understandings	Coefficient of Correlation	Number of Cases
Scores on *Motivation Inventory for Mathematics*	.514	116
Teacher ratings of students' motivation with regard to mathematics	.626	116
Scores on *Otis Quick-Scoring Test of Mental Ability*	.651	116
Scores on *Iowa Algebra Aptitude Test*	.722	116
Term grades in Algebra	.700	104
Term grades in General Mathematics	.510	86
Total Adjustment score of the *California Test of Personality*	.322	116

vation Inventory for Mathematics, and the types of achievement represented by the *Stanford Achievement Test*, term grades in algebra, and term grades in general mathematics, do not provide favorable evidence, because the correlations are smaller than those for English and Latin. If it were claimed that the *Motivation Inventory for Mathematics* provided a measure of *all* aspects of motivation which affect the manner in which the student applies himself to mathematical study, the findings would warrant serious questioning of this sub-hypothesis. This is not the case, however. To be comprehensive in this regard, the instrument would have to include measures of such factors as the extent to which the student is striving for high marks, the effects of the personal relationships between teacher and student on student effort, parental pressure for good marks, competition between students, and fear of failure in the course and of not graduating. These are important in terms of their effect on achievement, but they are *not* aspects of motivation with regard to mathematics which are the subject of this investigation, and, of course, are not measured by the instrument. Factors of this kind apparently dwarf the effects of factors measured by the *Motivation Inventory for Mathematics* on certain types of *immediate* achievement. It is believed that the effect of the latter type factors would be increasingly apparent over longer periods of time, in terms of success in elected courses in mathematics and creative achievement in mathematics, while the former would, in all likelihood, lose most of their positive effect upon the completion of required study. These considerations make it reasonable to retain for further testing the sub-hypothesis that motivation as measured by the *Motivation Inventory for Mathematics* does affect achievement in mathematics. Additional research involving longer periods of time in which more adequate controls are possible is needed.

With regard to the sub-hypothesis that there is a negative correlation between motivation with regard to mathematics and achievement in non-mathematical subjects, the statistically significant *positive* correlations in the findings indicate that factors such as motivation with regard to school work in general have a greater positive effect on achievement in non-mathematical subjects than the negative effect of the factors involved in the reasoning leading to the formulation of this sub-hypothesis. Further investigation is necessary to determine whether or not such negative effects can be identified with other factors adequately controlled.

The findings revealed no significantly high correlations between motivation with regard to mathematics and aspects of personality represented by part scores on the *California Test of Personality*. The correlation of .233 between scores on the *Motivation Inventory for Mathematics* and the Total Adjustment scores indicates that maladjustment in a student increases slightly the probability that the student will develop unfavorable rather than favorable attitudes toward mathematics.

It is important to note that the correlation of .700 between scores in mathematical understandings and term grades in algebra is slightly higher than the correlation between aptitude scores and term grades in algebra. This indicates that the test in mathematical understandings has considerable value in predicting algebra achievement. It also encourages the belief that teaching arithmetic with understanding pays dividends in terms of algebra achievement.

The correlations between motivation with regard to mathematics and other factors, as shown in Table 1, were not considered sufficiently high to warrant making statistical analyses for estimating indirectly the motivation with regard to mathematics of an individual from knowledge of scores on measures of these other factors.

Implications for Teaching and Curriculum Development

One of the most significant outcomes of the investigation is the instrument for measuring motivation with regard to mathematics. No instrument of its kind, which is comparable to it in comprehensiveness and ease of administration, was previously available. There are a wide variety of uses for the *Motivation Inventory for Mathematics* in educational and vocational guidance, classroom teaching, curriculum development, and educational research.

The theory of this investigation implies that teachers can inhibit the development of unfavorable motivation with regard to mathematics by preventing consistent experiencing of serious frustration by students in their study of mathematics, and can contribute to the development of favorable motivation with regard to mathematics by providing opportunity for satisfaction to result from the efforts of students. This requires taking more action in providing for individual differences than what seems to be the usual practice in our schools. It is impossible for all students to find adequate satisfaction in a classroom geared to the needs of the mythical "average" student, in which essentially the same performance and enthusiasm are expected from all students in uniform activities appropriate for the average student and centered upon uniform subject matter content prepared for the average student.

The extent to which satisfaction is experienced in mathematics classes is determined in large degree by the extent to which the mathematics curriculum is designed to provide for meeting actual needs, interests, and concerns of students. Therefore, further implications of the theory are that the curriculum should be planned in accordance with, among other considerations, the needs, interests, and concerns of students, and that day-to-day planning should be influenced by the students' expression of them.

The fact that knowledge of basic mathematical understandings was found to have a relatively high relationship to both motivation with regard to mathematics and other types of achievement in mathematics is evidence that it is important to teach mathematics with understanding, rather than for mastery of mechanical processes alone, as is common in practice.

The relationship between motivation with regard to mathematics and personal adjustment supports the contention that the responsibilities of the teacher are broader than simply being competent in the teaching of her subject. These findings indicate that it is important for a teacher to have some mastery of the principles of guidance and child development so that the student can be helped to achieve better adjustment and hence relieved of barriers to learning.

Implications for Further Research

Additional research needed on the instrument itself are: additional studies of its validity, simplification of the scoring system, and revision into a shorter form. Other kinds of needed research are: (1) devising an instrument for measuring motivation with regard to mathematics in which the student cannot influence his score by perceiving the "right" answers, (2) determining how motivational patterns with regard to mathematics develop, using approaches such as individual case analysis and experimental studies, as well as the test-statistical approach used in this investigation, (3) investigating what the needs of students are, particularly those which frequently are met, or can be met, through mathematical activity, (4) designing and testing instructional procedures based upon a theory such as the one used in this investigation, (5) making investigations of the effect of teaching arithmetic with understanding on the motivation with regard to mathematics of students, and (6) making use of the *Motivation Inventory for Mathematics*, in in-

vestigations of the effectiveness of instructional methods, to make it possible to determine the effects of the methods on the motivation as well as on the achievement of the student.

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ATTENDANCE RECORD OF THE ELEVENTH SUMMER MEETING

St. Olaf College, Northfield, Minnesota
August 20-23, 1951

State	*Members*	*Guests*	*State*	*Members*	*Guests*	*State*	*Members*	*Guests*
Arkansas	3		Nebraska	10		Wisconsin	36	8
California		1	N. Hampshire	1		Wyoming	1	1
Colorado	3	1	New Jersey	4	1	Alaska		1
Dist. of Col.	2	2	New Mexico	1	2	Canal Zone	1	
Georgia	1		New York	8	2	Honolulu		1
Illinois	43	21	North Dakota	5	3	Canada	2	3
Indiana	10	5	Ohio	9	3			
Iowa	21	8	Oklahoma	2	2	Totals	308	151
Kansas	9	4	Oregon	1				
Maryland	2	1	Pennsylvania	4		GRAND TOTAL		459
Massachusetts	1	1	South Dakota	2				
Michigan	14	6	Tennessee	1	1	Three members from West Virginia were not able to attend, but supported the meeting by registering.		
Minnesota	100	67	Utah	1				
Missouri	6	2	Virginia	1	2			
Montana	1		Washington	2	2			

Reports from the Affiliated Groups Program

JOHN R. MAYOR, *Chairman*
Committee on Affiliated Groups

THIRD DELEGATE ASSEMBLY

Preliminary plans for the Third Delegate Assembly to be held in Des Moines, in the spring of 1952 should be completed by December 15. Members and officers of the Affiliated Groups and other interested members of the National Council are urged to send to the Committee on Affiliated Groups their recommendations for the agenda of the Delegate Assembly as soon as possible. The agenda for the Second Delegate Assembly in Pittsburgh was made up from suggestions of Groups and National Council members and it is planned to follow a similar procedure this year. One of the basic purposes of the Affiliated Groups program is to provide through the Delegate Assembly a more direct means for members of the Council to have a voice in Council affairs.

The first two Delegate Assemblies were largely concerned with organizational plans and establishment of definite working relationships between the National Council and the Affiliated Groups. These relationships appear now to be worked out well enough for efficient operation and hence Affiliated Groups should be ready to move into new areas of activities mutually beneficial for the Groups and for the national organization.

Both of the Delegate Assemblies considered activities which might be carried out through the Affiliated Groups and which would provide services to all members of the National Council. Illustrative of these are the Speakers Bureau, the traveling exhibit, the exchange of Group publications, and the joint sponsorship of contests. As plans for these activities develop they will be reported in this section.

OHIO COUNCIL OF TEACHERS OF MATHEMATICS

The first Group to become affiliated with The National Council of Teachers of Mathematics since the Second Delegate Assembly is the Ohio Council of Teachers of Mathematics, which has been granted Charter No. 90. The new president of the organization is Ona Kraft, Collinwood High School, Cleveland. Miss Kraft is a former member of the Board of Directors of the National Council. The 173 members of the new organization, as reported by the secretary, are representatives of all levels of instruction.

An interesting four page program announced the First Annual Conference of the Ohio Council which was held in April in Columbus. The theme of this meeting was: "Let us so teach mathematics that it will be more real, more important and more useful in life, in business, in industry, and on the farm."

NEW GROUPS

Two Groups not previously listed in this section have sent inquiries about affiliation with the National Council, since the Second Delegate Assembly. One of these Groups is The Southern New England Mathematics Association, the membership of which includes teachers from seventeen independent boys preparatory schools. The second Group is in the Houston, Texas area and is made up of mathematics teachers at all levels. This latter Group reached the decision to take steps toward affiliation following the first Mathematics Institute sponsored by the University of Houston in July.

Sponsorship of Panel Discussions

Since the First Delegate Assembly held in Chicago in April, 1950 the Committee on Affiliated Groups has sponsored a panel discussion by representatives of the Affiliated Groups at each National Council meeting. These panel discussions have been held at the Madison, Gainesville, Pittsburgh, and Northfield meetings. The participants have been chosen from the geographical area of the meeting, with the exception of the Minnesota (summer 1951) meeting. On the panel at Northfield there were representatives of eight Groups from New Jersey to California. Thirty of the fifty-one Affiliated Groups have been represented in one or more of these panels.

A similar program is planned for the Stillwater meeting during the Christmas holidays. Invitations to serve on a panel have been sent to presidents of the statewide Affiliated Groups in Arizona, Arkansas, Colorado, Kansas, Nebraska, Oklahoma, Texas; the urban area Affiliated Groups in Dallas, Oklahoma City, Tulsa, and Wichita; and the Affiliated Group in Eastern Colorado.

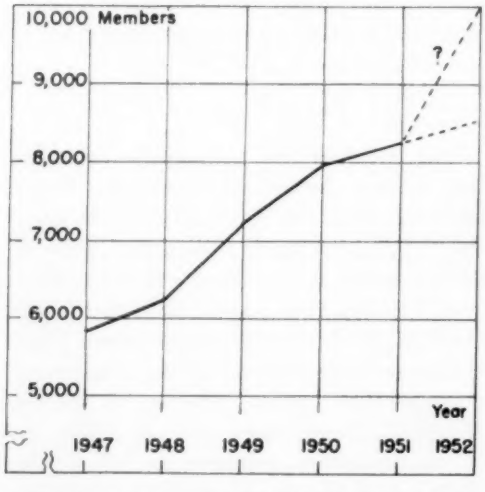
Membership Message

By Mary C. Rogers
Roosevelt Junior High School, Westfield, New Jersey

1951-1952 NCTM MEMBERSHIP SHOULD REACH A 10,000 TOTAL

This is the challenge facing your State Representatives and they, with your assistance and support, are doing a magnificent job meeting the challenge.

NCTM MEMBERSHIP GROWTH
1947–1951



During the year 1950–1951, thirty-four states have maintained their membership status or have shown substantial growth. Fifteen of these states have a record of continuous growth for the past four years.

NCTM membership totals are steadily increasing. Not only is growth currently persistent, but over the years there has been a steadily increasing rate of change. We have found the accompanying table and graph informative and helpful. We submit them to you for your study and consideration.

As has been our custom for the past two years, we shall again this year give special recognition to those schools with outstanding membership records—our 100% and "All but One" schools. We shall publish two listings of these Membership Honor Schools. The first will appear in the February, 1952 issue of The Mathematics Teacher. A second Honor listing, accompanied by a complete analysis of current membership status and periodic growth, is planned for the May, 1952 Teacher.

We need the assistance of each of you in locating these schools. Will you help us? Simply complete the accompanying form and mail it to

Mary C. Rogers
307 Prospect Street
Westfield, New Jersey

If your reports are in our hands by November 30, 1951, we shall guarantee the appearance of qualifying schools in our February, 1952 listing. May we look forward to hearing from a great many of you during the coming weeks.

Please accept our thanks for this assistance. Our best wishes to you for a most successful school year!

MEMBERSHIP REPORT TO THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

School.....
School Address.....
Number of Teachers of Mathematics in Your School.....
Number of National Council Members.....
Report Submitted by.....
Your Address.....
Names of Individual Members.....

MEMBERSHIP GROWTH BY STATES

States	Jan. 1947	Jan. 1948	March 1949	May 1950	May 1951	Average Change per period given	Net increase for four year period
Illinois	475	473	567	730	706	11.3%	48.6%
New York	513	643	644	721	689	8.3%	34.3%
Pennsylvania	412	438	478	520	634	11.5%	53.9%
Texas	200	208	224	360	405	21.2%	102.5%
Ohio	348	331	417	417	388	3.6%	11.5%
California	264	291	298	328	356	7.8%	34.8%
Indiana	137	318	318	318	333	34.2%	143.1%
New Jersey	277	312	325	350	318	3.9%	12.9%
Michigan	241	213	248	268	281	4.5%	16.6%
Massachusetts	229	238	250	266	274	4.7%	19.7%
Wisconsin	179	198	188	202	239	7.0%	33.5%
Virginia	114	123	187	201	244	22.2%	114.0%
Florida	102	130	161	179	220	21.4%	115.7%
Minnesota	138	144	179	204	209	11.3%	51.4%
Kansas	166	150	149	200	174	2.7%	4.8%
Missouri	133	122	146	176	168	6.9%	26.3%
Maryland	112	114	136	165	167	10.9%	40.2%
Iowa	129	107	144	171	163	7.9%	26.4%
Louisiana	80	100	153	142	162	21.2%	102.5%
North Carolina	103	96	158	137	161	15.7%	56.3%
Oklahoma	97	81	113	157	158	15.6%	62.9%
Connecticut	119	114	128	141	134	3.3%	12.6%
Tennessee	88	97	101	124	141	12.7%	60.2%
Washington, D.C.	105	132	120	128	110	2.3%	4.8%
Alabama	79	77	88	114	121	11.9%	53.2%
Georgia	40	67	71	99	113	31.8%	182.5%
Colorado	80	75	107	123	107	9.6%	33.4%
Nebraska	79	78	98	112	110	7.0%	26.6%
Arkansas	64	58	78	99	87	10.0%	35.9%
Mississippi	34	42	49	79	86	25.0%	152.9%
West Virginia	43	52	52	90	93	24.3%	116.3%
Washington	76	67	66	90	79	2.7%	3.9%
Kentucky	69	68	90	93	76	4.0%	10.1%
South Carolina	64	86	70	72	78	6.8%	21.9%
Oregon	77	80	84	65	75	0.4%	−2.6%
Delaware	23	24	24	31	44	18.9%	91.3%
New Mexico	18	42	30	32	43	36.5%	138.9%
Arizona	17	26	41	36	36	24.6%	111.8%
New Hampshire	21	31	27	30	38	18.1%	81.0%
Rhode Island	37	36	42	36	38	1.3%	2.7%
Utah	13	10	14	25	37	35.9%	184.6%
South Dakota	21	25	27	27	27	6.8%	28.6%
Maine	34	28	31	28	28	−4.2%	−17.6%
Montana	20	24	25	24	25	6.1%	25.0%
Wyoming	19	26	19	22	22	6.5%	6.5%
Vermont	16	27	25	21	22	12.6%	37.5%
North Dakota	19	15	18	19	20	2.5%	5.3%
Idaho	4	7	8	6	9	28.6%	125.0%
Nevada	4	10	8	9	6	27.3%	50.0%
Totals	5832	6254	7224	7987	8246	9.1%	41.4%

BOOK SECTION

Edited by JOSEPH J. STIPANOWICH

Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

HIGH SCHOOL

1. Algebra

A First Course in Algebra, 2nd Edition, by Walter W. Hart, formerly of the University of Wisconsin. Cloth, ix+389 pages, 1951. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. $2.28.

2. Second Course in Algebra

Everyday Algebra, Intermediate Course, by William Betz, Specialist in Mathematics, Rochester, N. Y. and Alfred P. Wincot, Hempstead High School, Hempstead, L. I., N. Y. Cloth, x+566 pages, 1951. Ginn and Company, Statler Building, Boston 17, Mass. $2.48.

A Second Course in Algebra, 2nd Ed., Enlarged, by Walter W. Hart, formerly of the University of Wisconsin. Cloth, viii+488 pages, 1951. D. C. Heath, 285 Columbus Avenue, Boston 16, Mass. $2.40.

3. Trigonometry

Trigonometry for Today, by Milton Brooks, and A. Clyde Schock, with Albert I. Oliver, Jr., Consultant. Cloth, ix+203+102 pages, 1951. Harper and Brothers, 49 E. 33rd Street, New York 16, N. Y. $2.96.

COLLEGE

1. 1st Year Mathematics

College Mathematics, A First Course, 2nd Ed., by W. W. Elliott, Duke University; and Edward R. C. Miles, Johns Hopkins University. Cloth, xii+436 pages, 1951. Prentice-Hall, Inc. Publishers, 70 Fifth Avenue, New York 11, N. Y. $4.85.

2. College Algebra

College Algebra, 2nd Edition, by Raymond W. Brink, University of Minnesota. Cloth, xvi+495 pages, 1951. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. $3.75.

Intermediate Algebra, 2nd Edition, by Raymond W. Brink, University of Minnesota. Cloth, xi+295 pages, 1951. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. $3.00.

3. Analytic Geometry and Calculus

Analytic Geometry and Calculus, by William R. Longley, Percey F. Smith, and Wallace A. Wilson, all of Yale University. Cloth, ix+578 pages, 1951. Ginn and Company, Statler Building, Boston 17, Mass. $5.00.

4. Statistics

Mathematics Essential for Elementary Statistics, Revised Edition, by Helen M. Walker, Teachers College, Columbia University. Cloth, xiii+381 pages, 1951. Henry Holt and Company, 257 Fourth Avenue, New York 10, N. Y. $2.75.

MISCELLANEOUS

Mary Everest Boole, by E. M. Cobham. Cloth, xiv+137 pages, 1951. The C. W. Daniel Company, Ltd., Essex, England. Available through the Open Court Publishing Company, La Salle, Illinois; the Concord Book Shop, Inc., 36 S. Michigan Avenue, Chicago 3, Illinois; and from the Lucis Publishing Co., 11 West 42nd Street, 32nd Floor, New York 18, N. Y. $2.00.

REVIEWS

Mathematics, A First Course, Myron F. Rosskopf, Harold D. Aten, and William D. Reeve. New York, McGraw-Hill Book Co., 1951. 476 pp., $2.60.

This book is designed for a first course in high school mathematics—not straight algebra, however, but an attempt to integrate the fundamental concepts of arithmetic, algebra, informal geometry, and numerical trigonometry.

A brief summary of the chapters can perhaps best indicate the content of the course: symbolism of algebra; fundamental operations with numbers; fundamental operations in algebra and very simple equations; geometry from the point of view of "Form, Size, Position," drawing and construction, and measurement (the chapter on measurement has some excellent material on the meaning of measurement and approximate numbers); formulas and their use in geometry; statistics and graphs (the part on statistics is unusual in a high school text—and is good); fundamental relationship; mathematics applied to personal and government finances, business, and aviation; indirect measurement by scale drawings and trigonometry; more simple algebra including positive and negative numbers, systems of equations, problem solving, factoring, power and roots (all with a tendency to avoid complicated cases); logarithms and slide rule.

About the presentation of this material: (1) The print and diagrams are particularly

clear and pleasing to the eye. (2) The explanations are so worded as to be interesting and meaningful to the student. (3) Whenever possible the authors bring out reasons for learning and mastering mathematical techniques. (4) Mathematics applied to everyday living is emphasized.

Some will object that the material in this book overlaps too much into eighth grade and tenth grade mathematics, and that there is not enough algebra. There are many school situations, however, in which this broader integration of basic concepts from various fields of mathematics is desirable. Every teacher, even though he may not want or be able to plan his course along the lines of this book, would find certain chapters (such as the ones on measurement, statistics, fundamental relationships) excellent reference material.—HENRY SWAIN, New Trier Township High School, Winnetka, Illinois.

Elementary Theory of Equations, Samuel Borofsky. New York, The Macmillan Company, 1950. x+302 pp., $4.25.

The traditional text in the theory of equations has had as its chief aim a study of the roots of algebraic equations. Borofsky states that it is the purpose of this volume to introduce the student to some of the concepts of modern algebra as well. The elementary concepts of field and polynomial over a field are used and the author states that it is hoped that the text will serve as a bridge between elementary algebra and some of the more abstract ideas which are a part of modern algebra.

The text is organized in such a manner that the first eight chapters may be considered as a brief but rather complete study of equations of higher order. Ruler and compass constructions are delayed until chapter nine. They are followed by the more abstract concepts of algebraic number fields and symmetric polynomials. A study of determinants and matrices completes the text.

The book is well organized, attractive in appearance, and is written in a clear and concise manner with sufficient problem material to fix the principles introduced.

The inclusion of material not ordinarily found in a text in theory of equations should do much to improve a course which seemed to lack practical considerations and was only too often of little interest to the students. In the opinion of the reviewer, this book is a welcome addition to those commonly used and could do much to revive interest in the course.—HERBERT HANNON, Western Michigan College of Education, Kalamazoo, Michigan.

Differential Equations, James E. Powell and Charles P. Wells. Boston, Ginn and Company, 1950. vi+205 pp., $3.00.

This is an excellent text for a three-semester hour first course in differential equations. It is intended for engineering and science students as well as mathematics majors. A few general methods are emphasized with special methods subordinated to these in an effort to combat the impression frequently left with the student that the solution of differential equations is a matter of learning numerous special and unconnected devices.

The exercises and applications are carefully selected and organized. The reviewer is especially pleased to see for the first time a mathematics text in which the authors deliberately include some problems that the student cannot solve.—R. A. BEAVER, New York State College of Teachers, Albany, New York.

An Introduction to the Theory of Statistics. G. Udny Yule and M. G. Kendall. Fourteenth Edition. New York, Hafner Publishing Co., 1950. xxiv+701 pp. $7.00.

The first edition of this work was published in 1911, the first systematic treatise on the then almost unknown subject of statistics. The theory of probability had reached a very satisfactory form by the end of the ninteenth century, since nearly a century of research had followed the publication in 1812 of Laplace's *Théorie analytique des probabilités*. But the application of this theory to data and the development of what we today call mathematical statistics has been almost entirely the product of the twentieth century. The treatise of Yule thus appeared near the beginning of the modern trend toward statistics. We should not be surprised, therefore, to see that it has reached fourteen editions. The eleventh edition, which appeared in 1937, was published in collaboration with M. G. Kendall, who adapted the older book to the rapidly growing subject of statistics.

The present volume is again a substantial revision. In the words of Kendall "The major additions fall into two groups. Chapters 21–23 expand the former treatment of small-sample theory and give an introduction to the practical problems of sampling. Chapter 25–27 give an account of index numbers and the elementary theory of time series. Chapter 13 on practical problems of correlation has also been re-written. Additions have been made in the remaining chapters to keep the treatment abreast of new discoveries, some of the examples have been modernized and some further exercises added."

This book should be on the shelf of anyone who is seriously studying statistics or who is engaged in applying the methods of this great science to practical data. Let us hope that the book goes into many more editions.—H. T. DAVIS, Northwestern University, Evanston, Illinois.

Contributions to Mathematical Statistics. R. A. Fisher. New York, John Wiley and Sons, Inc., 1950. 672 pp. $7.50.

This work has been prepared in order to make accessible from their many sources of publication the papers of the English statistician Ronaly Aylmer Fisher, more generally known to all statisticians as R. A. Fisher. The volume contains a portrait of Professor Fisher, a bib-

liography of the author, originally published in 1938, an editor's preface by Walter A. Shewhart, a preface by the author, an index prepared by John Tukey, and a facsimile reproduction of forty-three papers.

The reason for the present collection is explained by the editor as follows: "Like so many others, for more than a quarter of a century I have found inspiration and guidance in the original contributions of Professor Fisher. Some of my reprints of his early papers have been almost worn out through constant use, and for several years I have wished that I had a bound volume of his papers, particularly if he could be persuaded to preface each article with a brief note setting it in the proper perspective as of today. After some persuasion, Professor Fisher agreed in the fall of 1946 to undertake the selection of what he now considers to be his most outstanding contribution and to provide brief introductory notes wherever he felt that such information would be of help to students of his work. The present book is the result."

The volume is beautifully printed in large format ($11 \times 8\frac{1}{2}$ inches). The pages are not consecutively numbered, but retain the numbers of the pages in the original articles.—H. T. Davis, Northwestern University, Evanston, Illinois

Table of the Bessel Functions $Y_0(z)$ and $Y_1(z)$ for Complex Arguments. Prepared by the Computation Laboratory National Bureau of Standards. New York, Columbia University Press, 1950. xl+427 pages. $7.50.

This fine volume of tables, prepared under the direction of Arnold N. Lowan, is one of the most ambitious projects of the Computation Laboratory, which, originally organized in 1938, has been operated by the National Bureau of Standards since March, 1943. This is the twenty-fourth volume of the series, which is an enduring monument to the computational development of American mathematics. Throughout the preparation of these many volumes Dr. Lowan has been the guiding spirit of the laboratory and the highest credit should be given to him for the excellence and accuracy of the computations.

The present work is devoted to the evaluation of the two Bessel functions of second-kind over a complex domain $z = \rho e^{i\phi}$, where ρ ranges from 0 to 10 at intervals of 0.01 and ϕ ranges from 0° to 90° at intervals at 5°. The computation is carried out to 10 decimal places. Auxiliary tables of the functions $Y_0(z) - (2/\pi)J_0(z) \log_e \rho$ and $Y_1(z) - (2/\pi)J_1(z) \log_e \rho + (2/\pi z)$ and a table of the complex zeros of $Y_0(z)$, $Y_1(z)$, and $Y_1'(z)$ are also included.

A foreword by R. von Mises explains the importance of these functions in applied work and indicates the relationship of the tables to computation by high-speed calculators. In an extensive introduction the properties of the functions are described and graphs of their contour lines are included. The work is a notable contribution to the many recent tables of the Bessel functions.—H. T. Davis, Northwestern University, Evanston, Illinois.

Le Raisonnement Mathématique de l'Adolescent, Louis Johannot. Neuchatel, Switzerland, Delachaux and Niestlé, 1947. 174 pp., $2.00.

This volume is one of several in the series known as Actualités Pédagogiques et Psychologiques, published under the auspices of l'Institute des Sciences de l'Education de l'Université de Genève (Institute J. J. Rousseau). In essence it is an analysis of the psychology of learning in mathematics with reference to individuals from thirteen to eighteen years of age. There is a short preface by the well-known French psychologist and educator Jean Piaget, whose recent volume, *The Psychology of Intelligence*, has just been published in English by Harcourt, Brace (1950).

The Monograph in question is a commendable piece of original research. Part 1, a brief historical survey, is followed by the experimental evidence proper (Chapter 2, 3, 4 and 5). The data consist of a large number of individual pupil-interviews, intimately analyzing their thought processes when engaged in solving problems in arithmetic and algebra. Chapter 2 presents the evidence for the four steps in the development of mathematical reasoning: (1) understanding in terms of concrete manipulations (13 yr.); (2) understanding in terms of graphic representation (12 yr.–14 yr.); (3) understanding intuitively, or in formal arithmetical terms (13 yr.–17 yr.); (4) understanding in formal algebraic terms (17 yr. or over). In chapter 3 we find similar "clinical" evidence of learning difficulties with mathematical operations, notably the concept of inverse operations, as applied to multiplication and division of fractions. Chapter 4 deals in the same manner with the combination and order of operations, and the "psychological significance of algorithms." Chapter 5 deals with "abstract problems,"—generalizations. Here are discussed such considerations as the meaning of an equation and the recognition of relationships between quantities. Chapter 6 gives an overall view of the development of reasoning ability as maturation progresses, including the evolution of perception, sensorimotor intelligence, and intuition, tracing the growth of such concepts as inequality, identity, inverse, and the associative and commutative principles. The concluding section (Chapter 7) suggests certain pedagogical implications, the burden of which may be summed up as follows: comprehension always goes from the concrete to the general; meaning grows out of perception; ideas and generalizations are never "learned" *per se*, but are literally discovered; the learner can only generalize after he understands what he is doing.

A strong case is made for an abundance of laboratory work in teaching mathematics, together with continual emphasis on practical applications. Unless your French is extremely rusty, the book is quite readable, and deserves a place in the library of all who are interested in mathematical education.—William L. Schaaf, Brooklyn College, Brooklyn, New York.

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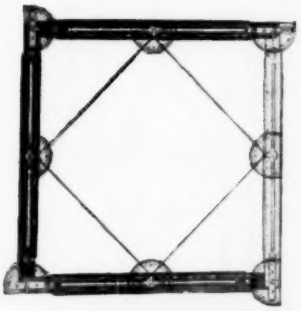

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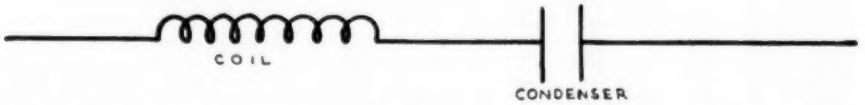


The reactance of a circuit measures the difficulty of passing AC (Alternating Current) through it. The smaller the reactance, the greater the current.

Reactance (X_L) of a coil

$$X_L = 2\pi f L$$

L is the inductance of the coil

f is the frequency (cycles per second) of the current

As f increases...
X_L increases

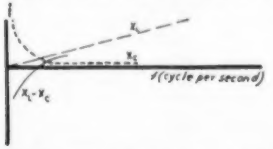
Reactance X_c of a condenser

$$X_c = \frac{1}{2\pi f C}$$

As f increases...
X_c approaches zero

At any frequency the reactance of the circuit at the top is the reactance of the coil minus the reactance of the condenser.

$$X_L - X_c = 2\pi f L - \frac{1}{2\pi f C}$$



At this frequency the reactance is zero, and we have "resonance". Current is maximum.

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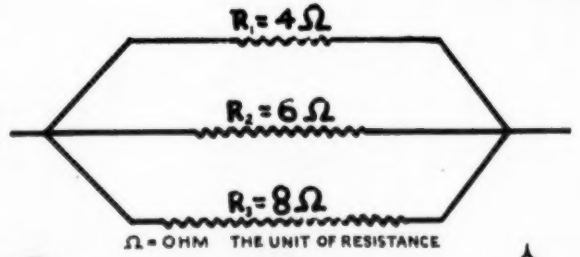
$$\frac{1}{x}=\frac{bc}{abc}+\frac{ac}{abc}+\frac{ab}{abc}$$

$$\frac{1}{x}=\frac{bc+ac+ab}{abc}$$

$$x=\frac{abc}{bc+ac+ab}$$



Ω = OHM THE UNIT OF RESISTANCE

The ELECTRICAL Problem

$$\frac{1}{R_{TOTAL}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$R_{TOTAL}=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$

$$=\frac{4\times6\times8}{6\times8+4\times8+4\times6}$$

$$=1.85-\Omega$$

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What is the value of X in this expression?

$$\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

$$\frac{1}{x}=\frac{b}{ab}+\frac{a}{ab}$$

$$\frac{1}{x}=\frac{b+a}{ab}$$

$$x=\frac{ab}{b+a}$$

If $a=2\times10^{-6}$ *and* $b=3\times10^{-6}$ *what does X equal?*

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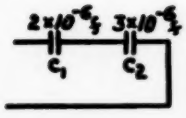

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

$$C=\frac{C_1\ C_2}{C_1+C_2}$$

$$C=\frac{2\times10^{-6}\cdot 3\times10^{-6}}{2\times10^{-6}+3\times10^{-6}}$$

$$C=\frac{6\times10^{-12}}{5\times10^{-6}}$$

$$C=1.2\times10^{-6}\text{ ƒ}$$

C = CAPACITY, MEASURED IN TERMS OF FARADS. A MICROFARAD (μƒ) IS ONE MILLIONTH OF A FARAD. EXPRESSED IN POWERS OF TEN, THE ENGINEER'S SHORTHAND, ONE MICROFARAD (.000001ƒ) IS 1×10^{-6}ƒ. $2\mu f=2\times10^{-6}$ƒ, AND $3\mu f=3\times10^{-6}$ƒ.

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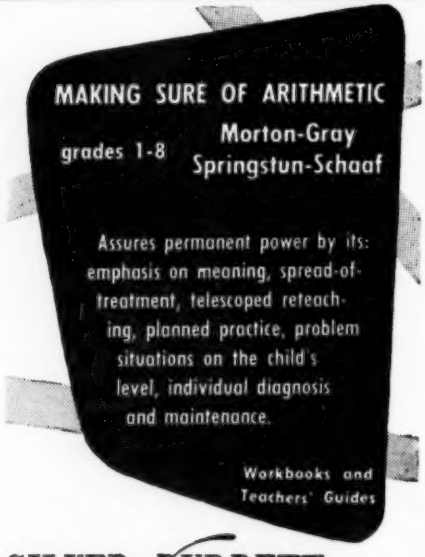
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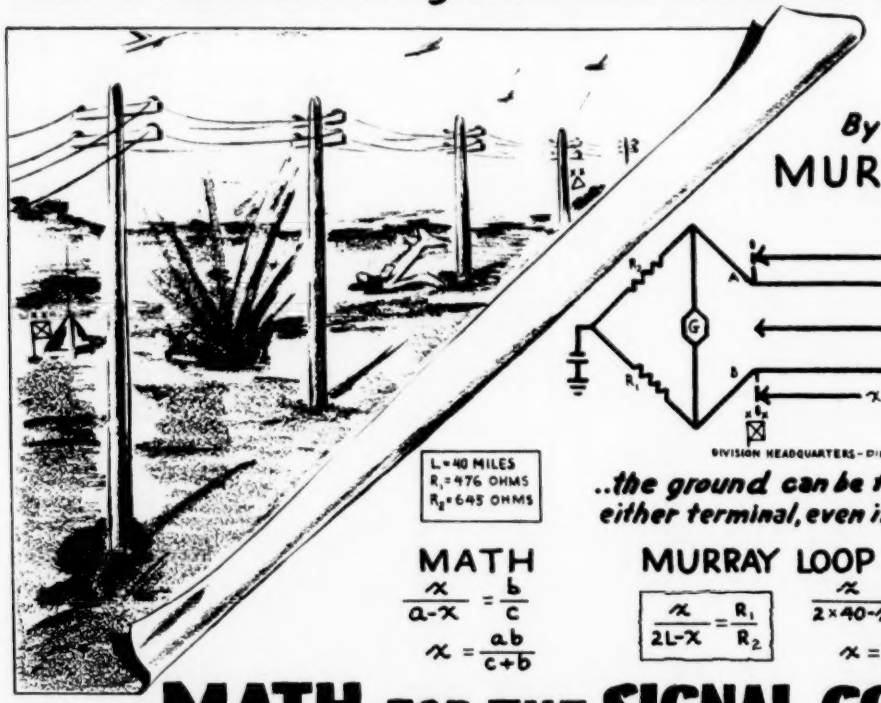

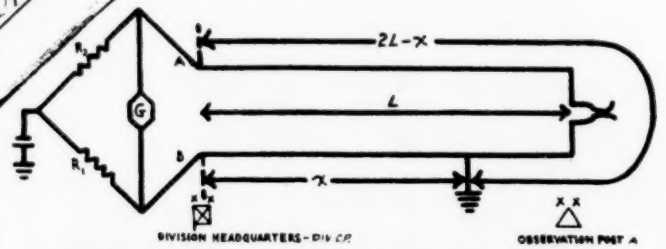

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MURRAY LOOP...

L = 40 MILES
R_1 = 476 OHMS
R_2 = 645 OHMS

..the ground can be found at once from either terminal, even if separated by miles.

MATH

$$\frac{x}{a-x} = \frac{b}{c}$$

$$x = \frac{ab}{c+b}$$

MURRAY LOOP COMPUTATION

$$\frac{x}{2L-x} = \frac{R_1}{R_2}$$

$$\frac{x}{2\times 40 - x} = \frac{476}{645}$$

$$x = \frac{80\times 476}{645+476} = 34\text{-Miles}$$

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WHAT IS GOING ON IN YOUR SCHOOL?

Edited by JOHN R. MAYOR *and* JOHN A. BROWN
The University of Wisconsin, Madison, Wisconsin

A TABLE summary of the first 15 replies to the questions on Mathematics Enrollments, published in this Department in the November, 1950 number of THE MATHEMATICS TEACHER was given in the April, 1951 number. In that number the questions were repeated with the hope that additional replies might be had. A similar summary of 14 additional replies received by June 15 is given at the end of this report.

The questions included in the three parts of this study are repeated for convenience in reading the summary of replies to part A, Mathematics Enrollments, and to provide one more opportunity for interested teachers to take part in this study.

Final summaries of replies to these parts of the study will be given in the April, 1952 number of THE MATHEMATICS TEACHER.

The questions are:

A. Mathematics Enrollments
1. How many years of sequential mathematics (in grades 9–12) is it possible for a graduate of your school to take?
2. What is the mathematics requirement for graduation from your high school?
3. How long has this requirement been in effect?
4. What per cent of your graduates of 1951 completed
   a. one year of mathematics in grades 9–12?
   b. two years of mathematics in grades 9–12?
   c. three years of mathematics in grades 9–12?
   d. four years of mathematics in grades 9–12?
5. What per cent of your graduates of 1951 entered college this fall?

B. General Mathematics
1. Does your school offer both algebra and general mathematics (or a course which might be called general mathematics) in the ninth grade? If not, does your school offer general mathematics in a later grade?
2. What per cent of your students take a course in general mathematics as their first course in high school mathematics (grades 9–12)?
3. For placement of students in general mathematics do you use:
   a. An algebra aptitude test?
   b. An arithmetic achievement test?
   c. The student's grade in eighth grade mathematics?
   d. An intelligence test score?
   e. Written recommendations of teachers?
   f. Other?
   Please indicate which of these or any other placement procedures you use.
4. Upon completion of one year of general mathematics may your students elect:
   a. A second year course in general mathematics?
   b. Algebra?
   c. Plane Geometry?
   d. Other courses in mathematics?
   e. None?
   If possible, indicate what per cent of these students enroll in each course.
5. Do the colleges which your graduates attend accept general mathematics on the same basis as they accept a course in algebra?

C. Third Year Mathematics
1. What courses do you offer for the third year of sequential mathematics?
   a. A third semester of algebra and a semester of solid geometry
   b. A third semester of algebra and a semester of trigonometry
   c. A second year of algebra
   d. A year of plane geometry
   e. Algebra and another subject in combination different from that in (a) or (b); if (e) is checked, please indicate also
      Number of weeks of algebra
      Number of weeks of solid geometry
      Number of weeks of trigonometry
   f. Other (please describe briefly).
2. Do you have ability grouping in your third year course?
3. Do you have a mathematics club:
   For juniors only?

For seniors only?
For both juniors and seniors?

4. Do you use films and film strips in the third year of sequential mathematics? If so, please list the two you have found most helpful.
5. Do your third year mathematics students use the library as part of the suggested study in this mathematics course?

Please send answers to these questions for your school to John R. Mayor, North Hall, Madison 6, Wisconsin by December 15, 1951.

In replying please give your name, the name and locality of your school and your high school enrollment. Also, please indicate the organization of your school such as 4-year high school, grades 9–12; senior high school, grades, 10–12; junior high school, grades 7–9; etc. The questions may be answered by number without use of a particular form or repetition of the questions. In reports no reference will be made to names of teachers or schools.

Report of Replies in Mathematics Enrollments

The 29 replies to part A which had been received by June 15 came from eighteen states. Twenty-three of the reports are from public schools and six from private schools.

While the number of replies is still much too small to provide an indication of general practices in American high schools it should be interesting for teachers in any school to compare their enrollments with those of 29 other schools, of course, recognizing differences in size and organization. The plan in the earlier report of listing schools in order of the per cent of graduates of 1950 entering college is repeated here.

It should be observed that 27 of the 29 schools offer four years of sequential mathematics. Another special characteristic of this group is that 4 schools require three years mathematics for graduation; 9 require two years of mathematics; 1 requires 1½ years; 9 require one year; 5 have no mathematics requirement; and 1 did not report on requirements. Readers will notice that three of those schools which require three years of mathematics for graduation had 100% of their graduates enter college.

The lengths of time that the present requirements have been in effect provide one of the most surprising aspects of the data in that for those schools reporting, only three indicated a change in requirement in less than ten years while several indicate that their present requirements have been in effect fifty or sixty years. It is likely that in those schools for which no answer to this question is reported there have been no recent changes.

(Continued on page 501)

Mathematics Enrollment in 14 Secondary Schools

(The numbers at the tops of the columns are the numbers of the questions above: the answers to questions 1, 2, and 3 are in years; the answers to questions 4a, 4b, 4c, 4d, and 5 are in per cents)

School	Grade Organizations	Enrollment	1	2	3	4a	4b	4c	4d	5
P	9–12	—	4	3	40	100	100	100	50	100
Q	8–12	250	4	3	60	100	100	100	80	100
R	9–12	—	4	1	15	100	90	20	10	70
S	9–12	1600	4	1	50	100	85	51	16	70
T	9–12	327	4	1½	5	100	100	100	78	61
U	7–9, 10–12	1300	4	0	18	81	64	50	11	45
V	9–12	—	4	1	15	96	88	35	9	43
W	9–12	365	4	0	—	95	82	47	15	42
X	9–12	1450	4	0	—	75	65	27	8	40
Y	9–12	340	4	2	—	100	100	14	0	36
Z	9–12	180	4	1	1	100	100	27	10	29
AA	9–12	545	4	1	20	100	50	33	4	20
AB	9–12	480	4	3	25	100	100	100	15	20
AC	9–12	—	3	2	50	100	100	38	0	13

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

32. *Cevians, Nedians, and Redians*

In the modern geometry of the triangle, a *cevian* is a line segment whose initial point is a vertex and whose terminal point is on the side opposite the vertex. The terminal point of the cevian divides the side of the triangle, either internally or externally, in a ratio which we may call the *terminal ratio*. For example, in Figure 1, AE, BF and CD constitute a *set of cevians* whose associated terminal ratios are $BE/EC=r$, $CF/FA=s$, and $AD/DB=t$.

Let us call the triangle PQR the *cevic triangle* of triangle ABC. The cevic triangle has its vertices the intersections of the cevians of a set. Furthermore, let triangle DEF, whose vertices are the terminal points, be known as the *menelaic triangle* of triangle ABC.

By analytic geometry—making use of the ratio formula, and the determinant form of the area of a triangle—it is readily shown that

$$(1)\quad \frac{\triangle PQR}{\triangle ABC}=\frac{(1-rst)^2}{(1+r+rs)(1+s+st)(1+t+tr)}$$

$$(2)\quad \frac{\triangle DEF}{\triangle ABC}=\frac{1+rst}{(1+r)(1+s)(1+t)}\,.$$

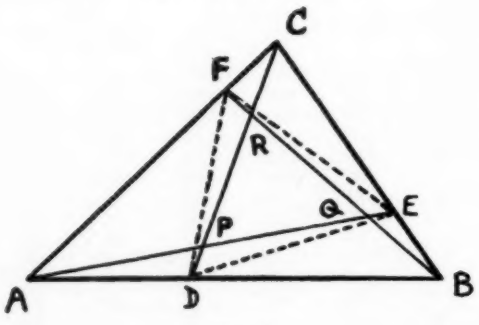

FIG. 1

From (1) and (2), by considering the conditions under which the areas of the cevic and menelaic triangles vanish, we obtain the following well-known theorems.

THEOREM 1. *Ceva's Theorem. In a triangle, if the three cevians of a set are concurrent, the product of their terminal ratios is 1.*

THEOREM 2. *Converse of Ceva's Theorem. If the product of the terminal ratios is 1, the cevians are concurrent.*

THEOREM 3. *Menelaus' Theorem. In a triangle, if the terminal points of a cevian set are collinear, the product of their terminal ratios is -1.*

THEOREM 4. *Converse of Menelaus' Theorem. If the product of the terminal ratios is -1, the terminal points are collinear.*

We note that when $r=s=t$ the cevians become *nedians* or *redians*, as discussed in THE MATHEMATICS TEACHER for January 1951 (vol. XLIV, pp. 46–48) and May 1951 (vol. XLIV, pp. 310–311). In this case formulas (1) and (2) above may be reduced to formulas (1) and (2) of page 310 by setting $r=p/q$.

By generalization, we may associate with any polygon its *cevic polygon* and its *menelaic polygon*. In particular, for a regular n-gon of area K, in which the cevians are all redians of ratio r, we may show that

(3) Area of Cevic Polygon

$$=\frac{\left(1+2r\cos\frac{2\pi}{n}\right)^2 K}{(1+r)^2+2r(1+r)\cos\frac{2\pi}{n}+r^2}$$

(4) Area of Menelaic Polygon

$$= \left[1 - \frac{4r \sin^2 \frac{\pi}{n}}{(1+r)^2} \right] K.$$

It is seen from (3) that the redians of a regular n-gon are concurrent if and only if $r = -\frac{1}{2} \sec 2\pi/n$. Also, by evaluating the limit of the right member of (3) as r increases without limit, it is found that the area S of the polygon bounded by the primary diagonals of a regular n-gon of area K is given by

$$(5) \qquad S = \frac{2 \cos^2 \frac{2\pi}{n}}{1 + \cos \frac{2\pi}{n}}.$$

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33. *More About Factoring*

With regard to *Miscellanea 15. A Method for the Speedy Factoring of the General Quadratic Trinomial* [THE MATHEMATICS TEACHER. Vol. XLII (December 1950), p. 421] may I report that the reverse operation to forming the binomial product $(ax+b)(cx+d)$ to give $acx^2+(ad+bc)x+bd$ has proved a most direct and reliable method in my Grade XI and XII classes.

Forming products and factoring are treated as inverse operations in a single chapter of the approved textbook for these grades of Algebra in Ontario. (The question of the factorability of the general trinomial, and factorizations where surds and complex quantities are needed are for the most part left for Grade XIII treatment.) After numerous and various binomial products are readily formed by inspection and the results given orally by the pupils, trinomials with small coefficients (say less than 15) in the second degree and absolute terms are factored by trial and error. The tediousness and inefficiency of this method is then pointed to for an example such as $24x^2-13x-60$, and so the question is naturally introduced: In a trinomial px^2+qx+r, how can we factor the product pr to give two numbers whose algebraic sum is q?

Once two such numbers are determined they may be taken as the values for ad and bc respectively in $acx^2+(ad+bc)x+bd$. The factoring may then be completed by reversing the steps one would take in multiplication. Thus:

$$\begin{aligned} px^2+qx+r &= acx^2+(ad+bc)x+bd \\ &= acx^2+adx+bcx+bd \\ &= ax(cx+d)+b(cx+d) \\ &= (ax+b)(cx+d) \end{aligned}$$

As a characteristic illustration, let us factor $24x^2-13x-60$. $24 \times 60 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \times 2 \cdot 2 \cdot 3 \cdot 5$. We have to re-group these to give a pair which *differ* by 13, the larger of them being the negative one. To do this with 100% accuracy is tedious but instructive when the method is first being used. The possible combinations are:

ad	bc
1	1440
2	720
3	480
4	360
5	288
6	240
8	180
9	160
10	144
12	120
15	96
16	90
18	80
20	72
24	60
30	48
32	45
36	40

The obvious pair is 32 and 45 with 45 negative.

We then have

$$24x^2+45x+32x-60$$
$$=3x(8x-15)+4(8x-15)$$
$$=(8x-15)(3x+4).$$

In examples such as this the pupils quite soon see roughly where to start in this systematic pairing of factors by noticing the size of the first degree term's coefficient.

As a second illustration of this method consider the second "doodler" in *Miscellanea 15:* What must be the missing coefficient in $18x^2 \pm Ax+54$ for it to be factorable? $18 \times 54 = 1 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Hence, for A to be prime to 18 and 54 as the problem required, its two parts must be $2 \cdot 2$ and $\cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, i.e. $A = \pm 247$.

A second proof of the lemma contained in the last sentence of the second paragraph of Case I of *Miscellanea 15* may also be of interest. This lemma stated: *When all three coefficients of the general quadratic trinomial are odd, it is not factorable in the domain of real integers.*

Since ax^2+bx+c has rational binomial factors if and only if its discriminant is a square, $(2p+1)x^2+(2q+1)x+(2r+1)$ where p, q, r, K are integers has binomial rational factors if and only if $(2q+1)^2 - 4(2p+1)(2r+1) = K^2$. Since the left hand side is odd, so is K^2 which we may then call $(2s+1)^2$. Therefore $(2q+1)^2 - (2s+1)^2 = 4(2p+1)(2r+1)$, i.e. the left hand side is 4 times an odd number. But the difference on the left hand side can be written $4(q+s+1)(q-s)$ and one of these last two brackets must be an even number. Thus the "absurdity" that a multiple of 8 is an odd multiple of 4, completes the "indirect" proof of the lemma.

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34. Some Nedian Details

In joining the discussion on nedians of a triangle[1] the writer wishes to adopt a slightly different definition of the nedian points. Let us say that a nedian point divides a side into two segments with ratio $n:1$. Thus, in figure 1,

$$V_2P_1 : P_1V_3 = V_3P_2 : P_2V_1$$
$$= V_1P_3 : P_3V_2 = n:1.$$

For purposes of comparison with the $p:q$ of Prof. Anning (The Mathematics Teacher [May 1951] p. 310) and the N of Prof. Satterly (The Mathematics Teacher [January 1951], p. 46) we have

$$n = p:q = (N-1)^{-1}.$$

The segment-to-segment ratio has, in this writer's experience, been more readily applicable than the side-to-segment ratio. Moreover, the basic relationships assume somewhat more simple expressions. The number n need not be integral, nor even rational. Interesting particular cases arise with n irrational.

Each nedian in a complete set of three is divided into three parts. In figure 1, call N_3V_1, N_2N_3 and P_1N_2 the *tip*, *middle*, and *stem* respectively of nedian P_1V_1. Then the equation

$$whole : tip : middle : stem$$
$$= (n^2+n+1) : (n+1) : (n^2-1) : 1$$

expresses the numerical relationships of these segments. Again in figure 1, notice four sorts of triangles—$V_1V_2V_3$, $V_1P_2N_3$ (3 such), $V_1V_2N_3$ (3 such), $N_1N_2N_3$. Let us refer to these as primary, corner, side, nedian respectively. Their areas have the ratios

$$primary : corner : side : nedian$$
$$= (n^2+n+1) : (n+1)^{-1} : n : (n-1)^2.$$

These results are easily obtained by elementary geometry (see reference 3 below).

In figure 2, let Q_1 be the intersection of lines V_1N_1 and V_2V_3; similarly Q_2 and Q_3. Then it is easy to show by means of Ceva's theorem that Q_1, Q_2, Q_3 are nedian points such that

[1] For earlier discussions of nedians and related concepts see *Miscellanea 18*, *26*, *32* in The Mathematics Teacher, vol. 44 (January 1951), pp. 46–48; (May 1951), pp. 310-312 and this issue, pp. 496–497.

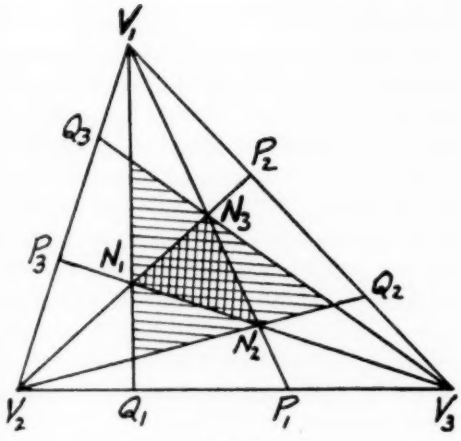

FIG. 2

$$V_3Q_1:Q_1V_2 = V_2Q_3:Q_3V_1 = V_1Q_2:Q_2V_3 = n^2:1.$$

The *tip*:*middle* ratio along the Q nedians is the same as the *stem*:*middle* ratio along the P nedians. The ratio of the areas of the respective nedian triangles is $(n+1)^2:(n^2-n+1) = \Delta V_1V_2V_3 : \Delta P_1P_2P_3$. Thus the relationship of the two nedian triangles duplicates that of triangles $V_1V_2V_3$ and $P_1P_2P_3$.

Let each side of a triangle be divided into a given odd number (say $2a+1$) of equal parts. The points of division nearest the midpoints are nedian points corresponding to $n=(a+1)/a$. The area of the nedian triangle is then an aliquot part of the area of the primary triangle. The number of times the latter contains the former is $(a+1)^3-a^3$.

Along the nedian the *middle* may be equal to *tip* or *stem*, but *tip* and *stem* can not be equal unless both vanish. The three segments may, however, be in arithmetic, geometric or harmonic sequence. For the geometric progression take $n=(1/2)(1+\sqrt{5})$. The nedian points then divide the respective sides in mean and extreme ratio. On the nedians, also, *tip*:*middle* = *middle*:*stem* $=(1/2)(1+\sqrt{5}):1$. It follows that *tip* = *middle* + *stem*. As to areas in this particular case, the nedian triangle is equal to each corner triangle. Figure 2 has $n=(1/2)(1+\sqrt{5})$. Then $n^2=(1/2)(3+\sqrt{5})$, the ratio for the Q points. For this ratio the nedian triangle is equal to each side triangle.

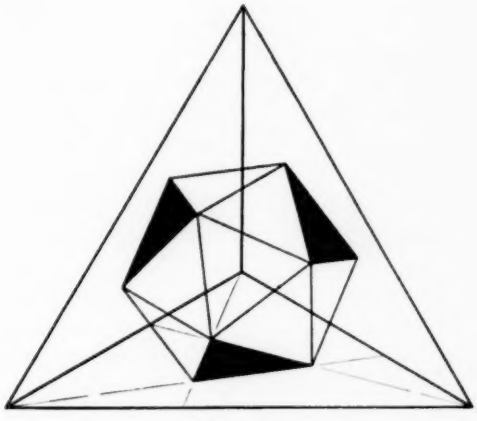

FIG. 3

A regular icosahedron can be inscribed in a regular tetrahedron so that each face of the latter contains a face of the former. To determine the position of a face of the icosahedron on a face of the tetrahedron construct in the latter a set of nedians corresponding to $n=(1/2)(1+\sqrt{5})$. The nedian triangle is the required face of the icosahedron. Figure 3 shows an orthographic projection of two such polyhedra. The icosahedron can be inscribed in the tetrahedron in two ways. The two positions correspond respectively to all "forward" nedians and all "backward" nedians.

In an equilateral triangle the nedians for $n=(1/2)(1+\sqrt{5})$ also furnish the elements necessary for constructing a model of the five interpenetrating regular tetrahedra which circumscribe a single regular icosahedron. Figure 4 shows a pattern for one of the twenty "points" of the model. To construct it divide sides AB and BC of an equilateral triangle ABC in mean and extreme ratio, as in Figure 4, and draw the corresponding nedians. These intersect at D. Draw BD. Extend AD to E, making $DE=DC$. Draw EC. Three faces such as $ABDEC$ surround each "point" of the model. Two adjoining "points" have broken edge $EDBE'$ on one coincident with $E'BDE$ on the other.

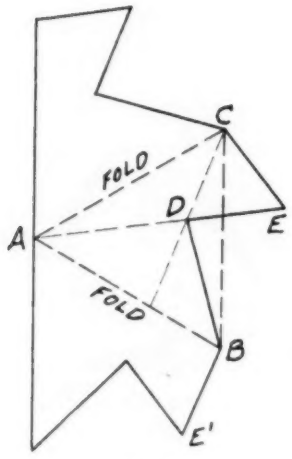

FIG. 4

A classified list of references is offered below.

GENERAL DISCUSSIONS

1. Alliston, N., *Mathematical Snack Bar.* Cambridge (England): W. Heffer and Sons, Ltd., 1936; pp. 11–13.

2. Dohner, H. I., "On a Problem of Steinhaus," *School Science and Mathematics*, vol. 41 (Nov. 1941), pp. 765–767.

3. Struyk, A., "Some Additional Notes on a Problem of Steinhaus," *School Science and Mathematics*, vol. 42 (Apr. 1942), pp. 325–330.

4. *School Science and Mathematics*, vol. 40 (May 1940), pp. 483–485, problem 1650.

5. *School Science and Mathematics*, vol. 41 (Nov. 1941), pp. 788–789, problem 1718.

PARTICULAR *n*

6. *School Science and Mathematics*, vol. 50 (Oct. 1950) p. 581, problem 2195.

7. *The Pentagon*, vol. 7 (Spring 1948) pp. 88–89, problem 3. (Generalized).

8. *School Science and Mathematics*, vol. 43 (Oct. 1943) pp. 684–685, problem 1821.

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10. Steinhaus, H., *Mathematical Snapshots.* New York: Oxford University Press, 1950, p. 8.

11. Steinhaus, H., *Mathematical Snapshots.* New York: G. E. Stechert and Co., 1938. pp. 7–8.

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12. Johnson, R. A., *Modern Geometry.* Boston: Houghton Mifflin Co., 1929, p. 175, p. 284.

13. *American Mathematical Monthly*, vol. 56 (Apr. 1949) pp. 269–270, problem 4260.

14. *School Science and Mathematics*, vol. 38 (Nov. 1938) pp. 935–936, problem 1556.

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35. Le Mathématicien Polycéphale

"Le mathématicien polycéphale" is the title applied to Nicolas Bourbaki, a mathematical personage of rapidly growing fame, by André Delachet in *L'Analyse Mathématique*, a recent pocket-book history of analysis.[2] The footnote to an article by Bourbaki in the *American Mathematical Monthly* states "Professor N. Bourbaki, formerly of the Royal Poldavian Academy, now residing in Nancy, France is the author of a comprehensive treatise of modern mathematics, in course of publication . . ."[3]

Both the adjective "multiheaded" and the "Royal Poldavian Academy" recall the fun that went with a grape-vine notice that Bourbaki was to speak at a section meeting of the American Mathematical Society's joint meeting with the National Council at Christmas-time in Columbus a few years ago. The fun lies in the fact that N. Bourbaki is not one man but a group of mathematicians, chiefly French, who have merged their mathematical personalities to produce in serial form an extremely significant work, *Eléments de Mathématique*, of which twelve volumes, the first dated 1939, the last 1951, have been published so far by Herman & Cie. as a part of the series *Actualités Scientifiques et Industrielles.*

Delachet regards their work as so significant that he devotes the concluding section of the last chapter of his history to "L'Ecole Bourbaki." He lists the chief members of this group as MM. Henri Cartan, Dieudonné, Chabauty, Ehresmann,

[2] André Delachet, *L'Analyse Mathématique.* (Paris: Presses Universitaires de France, 1949). This is booklet number 378 in the series *"Que Sais-Je?"* which was noted in this column in THE MATHEMATICS TEACHER, vol. 42 (Oct. 1949) p. 307.

[3] Nicholas Bourbaki, "The Architecture of Mathematics," *The American Mathematical Monthly*, 57 (April 1950) pp. 221–232. This article is the source of the phrases which we have enclosed in quotation marks. It, in its turn, is a translation by Arnold Dresden of the second of an extremely interesting collection of essays by diverse authors edited by F. Le Lionnais under the title *Les Grands Courants de la Pensée Mathématique.* (Cahiers du Sud, 1948).

Samuel, and Godement with the addition of MM. Weil and Brelot as participants who preserve their autonomy.

There are many interesting aspects of Bourbaki's work. Foremost perhaps is its objective which in its early chapters is to start from the beginning and to give a solid foundation for the whole of modern mathematics. He even says that to read his work from the beginning one would theoretically need to know no mathematics, but he admits that one would need a certain propensity for reasoning mathematically and a certain power of abstraction. This latter is particularly true since the approach is axiomatic and abstract, proceeding from the general to the particular. This is in line with his objective which may be further described as an attempt to show the structure of mathematics in such a way as to display its unity. This is opposed to the feelings of some persons that modern mathematics shows a "progressive splintering" manifest in its recent "exuberant proliferation."

Bourbaki finds a unifying concept not in "deductive reasoning" which is merely a method common to many areas, but in the "*axiomatic method.*" He regards the essential aspect of the axiomatic method to be not its use of syllogisms but its systematic study of the relations existing between different mathematical theories. This it does by means of searching out and analyzing the relatively few basic *structures* which are the common property of many mathematical systems. Examples of these are *algebraic structures* distinguished by a central theme of the laws of composition of abstract elements, *order structures*, *topological structures* dealing with the concepts of neighborhood, limit and continuity.

Recognition of these basic structures, Bourbaki feels, not only effects considerable economy of thought, but also provides a basis for that *intuition* which today more than ever plays a fundamental role in mathematical discovery.

Some of the mechanical aspects of Bourbaki's books are also of interest for their connection with his program. For example, many volumes include a dictionary not only of the terms used in the book but of all equivalent or related terms used in the chief languages. Bourbaki's terms have been selected and defined with care and deliberation. Where it will be particularly useful to readers to have certain axioms, and definitions before their eyes as they read these axioms and terms have been reproduced on a folded sheet which is then pasted in at the back of the book and listed in the table of contents.

Altogether "N. Bourbaki" stands for an extremely interesting and significant cooperative current international mathematical project.

What Is Going On In Your School?

(*Continued from page 495*)

Some interest may be found in the average of the per cents of the graduates of these 29 schools who had completed one, two, three, and four years of high school mathematics. These averages for the 29 schools are: 97% for column 4a, those who completed one year of mathematics in grades 9–12; 86% for column 4b, those who completed two years of mathematics in grades 9–12; 43% for column 4c, those who completed three years of mathematics in grades 9–12; 18% for column 4d, those who completed four years of mathematics in grades 9–12; 47% for column 5, per cents of 1950 graduates who entered college.

These averages may be fairly representative of schools from which about one-half of the graduates enter college but not of schools sending a lower per cent to college.

APPLICATIONS

Edited by SHELDON S. MYERS
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Your department editor has been doing a great deal of thinking and reading concerning the role of applications in mathematics instruction. Following are several key references which we have found: "The Necessary Redirection of Mathematics, Including its Relation to National Defense," by William Betz in THE MATHEMATICS TEACHER, April, 1942, page 147; "The Use of Applications for Instructional Purposes," by Edwin G. Olds in THE MATHEMATICS TEACHER, February, 1941, page 78; "The Mathematics Most Used in the Sciences of Physics, Chemistry, Engineering and Higher Mathematics," by George H. Nickle in THE MATHEMATICS TEACHER, February, 1942, page 77; *A Study of Problem Material in High School Algebra* by Jesse Powell, Contributions to Education, No. 405, Teachers College, Columbia University, 1929.

In the first reference, we agree with Betz that the organizing principle of mathematics instruction must be the interrelated system of mathematical concepts and processes. Betz emphasized, however, that, with the mathematical system as the center of orientation, experiences significant to children can be designed to provide significance, meaning, or both. Buckingham distinguishes between social significance and mathematical meaning. Applications may contribute to one, or the other, or both.

The study by Powell showed that teachers were unreliable judges in rating problems of interest to children. Pupils rated high verbal algebra problems involving the puzzle element, and boy and girl activities, but rated low problems of finance and those having a work theme. Teachers rated problems of finance or business high and problems with the puzzle element low. Difficulty did not seem to lower pupil ratings.

The foregoing references have been kept in mind in the development of this department. Most of the applications appearing in this department have been used in the classroom by the editor at the indicated grade levels and have proved successful in motivation and learning.

Ar. 11 Gr. 6–9 *Boys Entered in Soap Box Derbies Sometimes Have Real Arithmetic Problems.*

A boy and his racer cannot exceed 250 pounds by soap box derby rules. When a boy and his racer weigh less than 250 pounds, the boy will add weight, usually washers, so that the total will come up to the 250 pound maximum. He does this to take advantage of the pull of gravity. When he and his racer exceed 250 pounds, he must eliminate weight from his racer, since he cannot reduce that fast. Following are five boys with the weights of themselves and their racers. The washers which each boy adds or removes from his racer weigh one-half pound each. What must each boy do so that the combined weight of himself and his racer will qualify under derby rules?

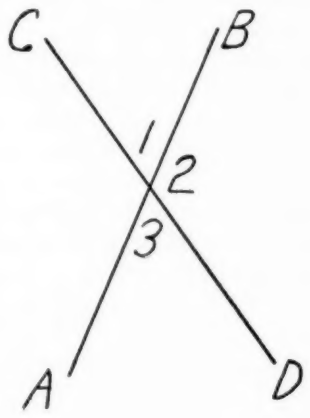
1. John Beekman 119½ lbs.; Blue Star 130½ lbs.
2. Jim Bailey 110¼ lbs.; Comet 134⅛ lbs.
3. Dick Manley 112½ lbs.; Super Jet 140¼ lbs.
4. Skippy Bedford 95 lbs.; Silver Streak 125¾ lbs.
5. Henry Jones 105⅛ lbs.; Red Rocket 126¼ lbs.

P. 4 Gr. 10–12 *A Logical Isomorphism in Plane Geometry*

Recently I examined an M.A. thesis in mathematics entitled *Some Applications of the Principles of Isomorphism and Variation to the Teaching of Geometry* by Ralph Mock, University of New Mexico, 1938. The first part of the thesis deals with an extensive collection of theorems and their proofs. Each theorem has an associated, non-geometric problem whose logical relationships and parallel solution are structurally identical with the geometric theorem but whose context and elements are different. In most cases the isomorphic identity was partial, not complete. The theorems thus illustrated ranged over polygons, including the Pythagorean Theorem, and circles. Here is a simple example of one of the theorems with its associated problem.

Geometric Theorem

If two straight lines intersect, the vertical angles thus formed are equal.



Associated Problem

A merchant at a trading post offers an Indian the following trade:

1. A box of cartridges for a bracelet and ring.
2. A box of cartridges for a bracelet and string of beads.

What is the comparative value of the ring and beads?

Correspondence:

The box of cartridges corresponds to 180°.
The bracelet corresponds to angle 2.
The ring corresponds to angle 1.
The string of beads corresponds to angle 3.

Solution: *Statements* — *Reasons*

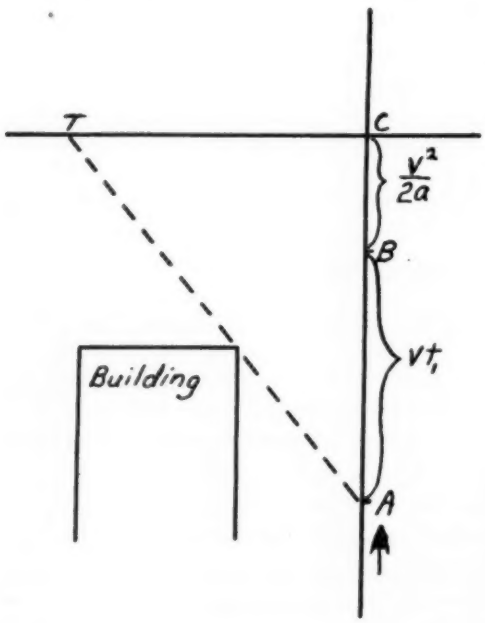
1. $\angle 1+\angle 2=180°$ — 1. The ring and bracelet are worth a box of cartridges.
2. $\angle 3+\angle 2=180°$ — 2. The string of beads and bracelet are worth a box of cartridges.
3. $\angle 1+\angle 2=\angle 3+\angle 2$ — 3. The ring and bracelet are worth the string of beads and bracelet.
4. $\angle 1=\angle 3$ — 4. The ring is worth the string of beads.

The associated problem for the Pythagorean Theorem involved the 1930 populations of nine counties in New Mexico.

The associated problems in this thesis were designed by the author to fit the corresponding geometry theorem. The thesis does not develop the technique for constructing such problems. However, with a little experience and insight, most teachers should be able to make up an indefinite number of associated problems using data and relationships dealing with the school itself. It must be stressed that in these analogues the social significance and context is irrelevant to the basic logical patterns.

C. Al. 3 Gr. 10–12 *A Grade Crossing Quadratic*

Dr. J. B. Coleman, formerly with the South Carolina Highway Department, wrote me last spring that he had had a number of requests for a "Grade Crossing Quadratic" which he had submitted in a paper to the Southeastern Section of the Mathematical Association of America last April. He sent me his brief paper, which had been abstracted in the *American Mathematical Monthly*, December, 1950, page 704. Below is his derivation of the formula.



Our problem is to find the velocity at which an automobile may approach a crossing where visibility is limited, so that upon seeing danger, such as an approaching train, the driver may be able to stop before reaching the crossing.

Suppose the driver, proceeding north with a velocity, v, reaches a point, A, in the figure before seeing that it will be necessary to stop short of the crossing, C.

The total reaction time for perceiving and acting varies with individuals and is generally betweeen .5 and 1 second. Calling the reaction time, t_1, the car will travel a distance of vt_1, or AB in the figure, before the brakes are applied. Perhaps it has never occurred to many drivers that, after the necessity for stopping is seen, the car with a speed of 50 m.p.h. will ordinarily travel a distance of 50 to 75 feet before the brakes can be applied.

After applying the brakes, we postulate uniformly negative acceleration, or deceleration, until the car speed is reduced to zero. With deceleration, a, and initial velocity, v, the distance travelled before coming to rest, s or BC in the figure, is related by the formula of elementary physics, $v^2=2as$. Hence, $BC=s=v^2/2a$. (The value of a will depend upon the character and condition of the road, of the tires, and of the brakes. For dry concrete with good tires and brakes, a may be about $.6g$, where g is the acceleration of gravity of 32 feet per second each second. For wet asphalt, poor tires and brakes, a may be $.2g$, or less.[1]

Calling AC, s_0, then $s_0=AB+BC$. Substituting the values found above, this equation becomes:

$$s_0=vt_1+v^2/2a.$$

Given the conditions of s_0, t_1, and a, the positive root of this equation, solved for v gives the maximum speed allowable under the given conditions. The student will get an appreciation of the formula only by applying it with different values of the conditions. This will serve the purpose of comparison, as well as information. He may check the value of v dimensionally, and will need to exercise care dimensionally as regards feet per second and miles per hour, since the results would ordinarily be given in miles per hour. (In the formula given below in the footnote, t_1 and a have been so modified that r is in miles per hour and d is in feet.)

[1] The Seventeenth Yearbook, *A Source Book of Mathematical Applications*, of The National Council of Teachers of Mathematics, on page 95, assumes an a of 22 feet per second per second and a t_1 of .75 seconds in deriving the formula, $d=.045\ r^2+1.1\ r$, for minimum stopping distance.

The equation might be termed a "live quadratic" in that the coefficients have physical significance. Such coefficients are called "parameters." It should serve a useful purpose if it does nothing more than direct the attention of some of our youthful drivers to the role played by reaction time, in a quantitative way.

C. Al. 4 Gr. 10–12 *An Inductive-deductive Experiment with the Tower of Hanoi Puzzle*

The Tower of Hanoi Puzzle can provide the basis for a neatly controlled inductive-deductive experiment in algebra which has many of the characteristics of scientific method. A description and some of the theory of this puzzle can be found in such books as *Mathematical Recreations* by Maurice Kraitchik, W. W. Norton, Inc., pp. 91–93 and *Riddles in Mathematics* by Eugene Northrop, D. Van Nostrand Co., pp. 25–27. Students can make them easily out of wood in the shop. The puzzle consists of three pegs set in a wood base and five to seven, round, or square, discs with holes in the center and varying gradually in size, so that, when placed over a peg in order of size, they look like a small pyramid. The object of the puzzle is to transfer all the discs from one peg to another by moving one disc at a time and by not placing a larger disc on a smaller one. The idea is to accomplish this with a minimum number of moves. For purposes of trying this application, you can cut the discs as squares from cardboard and imagine the three pegs.

When students attempt this puzzle with 3, then 4, then 5 discs, they realize that the minimum number of moves each time jumps accordingly. It is easy to get them to pose the research problem of finding the functional relationship between number of discs and minimum number of moves. It is interesting to see these young people naturally begin by trial and error to gather experimentally the following table of data:

n	m
number of discs	minimum moves
1	1
2	3
3	7
4	15
5	31
n	?

Soon they restate the problem as follows: What uniform set of operations, if any, can be performed on the left numbers so as to produce the right numbers correspondingly?

Some students begin to take differences between the values of m and discover that as far as they go down the table the first set of differences is a geometric progression with common ratio of 2. They also discover that the differences among these differences (second order differences) are likewise a similar geometric progression and that this is true for the third order differences. They examine the literature and find that if the first order differences are constant, the function is linear; if the second order differences are constant, the function is second degree, and so on. They also find that if successive order differences remain progressions, the function is some kind of exponential function. Before long after this, some discover that the expression $2^n - 1 = m$ correctly relates n and m for the first five cases. They have succeeded in making an inductive generalization from data.

Almost immediately someone will inquire whether this formula will accurately predict the minimum moves for 6 discs. The students then apply the formula deductively for the specific case of 6: $2^6 - 1 = 63$ moves. The cycle is finally completed when the students experimentally verify this prediction with the puzzle.

Usually advanced classes are smaller in number. With puzzles immediately available for groups of two students each, the foregoing experiment can be successfully accomplished in a 50 minute period, providing the students do not have to check too long on the theory of differences.

MATHEMATICAL RECREATIONS

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SOME years ago much ado was raised concerning the indirect method of proof in Geometry. The readers of THE MATHEMATICS TEACHER may still find great pleasure and derive much profit from reading the chapter on this topic in the FIFTH YEARBOOK OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS written by Professor Clifford B. Upton. In some quarters, however, some doubts have been expressed concerning the efficacy of the indirect method of proof.

This department does not propose to take sides in any controversy concerning the advisability of using or of discarding the indirect method of proof in classroom instruction. However, this department is of the opinion that this method of proof, as well as the processes which are employed in it, need not be confined to geometry exclusively. As a matter of fact, the indirect method may be employed with considerable effect in situations other than geometric. Furthermore, many mathematical recreations utilize the indirect method and with considerable effect, to be sure.

Here are a few problems which could have been probably located in the files of any Government agency which is concerned with the enforcement of the law.

A suspicious character was arrested and the arresting officer found in his possession a bag containing 80 silver dollars. Upon the examination of these coins the arresting officer could not be certain whether they were counterfeit. His tip was that one of the coins was lighter in weight than the remaining 79. He did not have a weighing machine with him. However, he constructed a balanced weighing scale and in four steps he extracted the counterfeit coin. How did he do it?

He divided the coins into three groups. Two groups contained 27 coins each. The third group contained 26 coins.

First, he weighed the two groups with 27 coins each by placing 27 coins on the pans of the balance scale. If the scale was balanced, then the lighter coin was located in the group with the 26 coins. If the scale did not balance, then the lighter coin was located in the group of 27 coins which was lighter.

Once the group in which the lighter coin was located, he put aside the other two groups of coins, and he worked on the smaller group (which could have contained either 26 or 27 coins, as indicated in the preceding paragraph). He subdivided this new group into three new groups of coins. If the group contained 27 coins then each new group contained 9 coins. If the group contained 26 coins, then he had two groups containing 9 coins each and one group containing 8 coins.

He balanced two groups of 9 coins each. This enabled him to determine the group which contained the lighter coin. Thus he finally obtained a group with 9 or 8 coins. Note that his method in this case was the same as the method he employed when he started his experiments.

The last group of 9 or 8 coins was again subdivided into three groups. If there were 9 coins in the group, then each new group contained 3 coins each. If there were 8 coins in the group, then two groups contained 3 coins each, and one group contained 2 coins.

Following the same method he ascertained in which the three new groups the lighter coin was located.

Finally, he was left with either 3 or 2 coins. Applying his method once more he

succeeded in separating the lighter coin.

The readers will note that the method of separating the lighter coin is indirect in nature. This type of a problem could be presented as a puzzler in the ninth grade.

Suppose that there are 12 coins and it is known that one of them is counterfeit. But it is not known whether this counterfeit coin is lighter or heavier than a real coin. The separation of the counterfeit coin may be accomplished in three steps. Only balanced scales without weights are allowed.

The 12 coins are divided into three groups of four coins each. When we place 4 coins on each pan of the balance scale two possible situations may take place.

A. The scale will be balanced.

B. One of the pans of the scale will be heavier.

A. If the scale is balanced, then the counterfeit coin is located in the remaining group of four coins, while the 8 coins which were weighed are all real. Take three coins from the group in which the counterfeit coin is located and place them on one pan of the scale. Balance these three coins with three coins which are known to be real. If the scale is balanced then the remaining fourth coin is the counterfeit. If the scale is not balanced then we know that the counterfeit coin is among the three coins, and we also ascertain whether it is lighter or heavier. The determination which of the three coins is the counterfeit is accomplished according to the method which was described above.

B. If one of the pans of the scale is heavier, we immediately know that the counterfeit coin is among the 8 coins which were weighed. But we do not know whether it is heavier or lighter than a real coin.

Label the coins on the heavier pan by numbers 1, 2, 3, and 4, and the coins on the lighter pan by 5, 6, 7, and 8.

Place coins 1, 2, and 5 on one pan, and the coins 3, 4, and 6 on the other pan. If the scale is balanced, then we know that either coin 7 or coin 8 is counterfeit. The determination of the counterfeit coin is accomplished by comparing (weighing) one of them with a coin which is known to be real.

If the pan with the coins 1, 2, and 5 is heavier, then we know that the coins 3, 4, and 5 are real. This conclusion is based on the fact that prior to the second weighing we exchanged the places of the coins with respect to the coins 1 and 2, and thus the pan on which the coins 1 and 2 are located should be lighter. But this is not observed. Thus, one of the coins 1, 2, or 6 is counterfeit. If the coin 6 is counterfeit, then it is lighter than a real coin. If one of the coins 1 or 2 is counterfeit, then the counterfeit coin is heavier than a real coin. Place coin 1 on one pan and coin 2 on the other pan of the scale. If the scale is balanced, then coin 6 is counterfeit. If the scale is not balanced, then the heavier coin is counterfeit.

If the pan with the coins 3, 4, and 6 is heavier, then, by analogy we conclude that coins 1, 2, and 6 are real, and if coin 5 is counterfeit, then it is lighter than a real coin. If coin 3 or coin 4 is counterfeit, then the counterfeit coin is heavier than a real one. Place coin 3 on one pan of the scale and coin 4 on the other pan of the scale. If the scale is balanced then coin 6 is counterfeit. If the scale does not balance then the heavier coin is counterfeit.

The above problems may be generalized, and this generalization should not offer any undue difficulties to a pupil who studies intermediate algebra.

We have n coins. One of these coins is counterfeit, and it is known that it is lighter than a real coin. Determine the minimal number k of the consecutive weighings on a balance scale (the weighing is done without any weights) which will lead to the determination of the counterfeit coin. All the real coins are of the same weight.

10,052 NCTM Members in 1952?

AIDS TO TEACHING

Edited by

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BOOKLETS

B. 67—Money Management Booklets

B. 68—Better Buymanship Booklets

Household Finance Corporation, 919 North Michigan Avenue, Chicago, Ill.

Booklets; 6″×9″; $.05 each.

Description of B. 67: There are eleven booklets in the Money Management Series of the following topics: Your Budget, Children's Spending, The Health Dollar, The Food Dollar, Your Clothing Dollar, The Shelter Dollar, Home Furnishings, the Recreation Dollar, Your Shopping Dollar, Time Management, and How to Buy Life Insurance. They are colorfully illustrated, immensely practical, and filled with charts, tables and pictures which serve as excellent motivation.

Description of B. 68: There are eighteen booklets in the Better Buymanship Series which discuss the points to watch for when purchasing certain products. The subjects covered are these: Meat, Fish, Poultry and Eggs; Fresh Fruits and Vegetables; Processed Fruits and Vegetables; Dairy Products; Playthings; Cosmetics; Soap and Other Detergents; Shoes; Hosiery; Fabrics; Furs; Household Textiles; Floor Coverings; Furniture; Dinnerware; Kitchen Utensils; Vacuum Cleaners; and Home Heating.

Appraisal of B. 67 and B. 68: This is the best set of inexpensive booklets on consumer education available at a price low enough so that sets can be given to individual pupils. The Money Management set should be in the classroom in quantity so that each pupil can have one of each in the set. Among these it may be well to buy enough each year so that pupils can have their own personal copies of the one called "Your Budget." This has blank pages which can be filled in as exercises while the class discusses the budget. The other volumes in the Money Management set contribute useful information, but they can be turned in to the teacher at the end of the year and used over again.

One complete set of the Better Buymanship Series for each class would be sufficient for reference purposes.

B. 69—Facts You Should Know about Borrowing

B. 70—Facts You Should Know about Securities

B. 71—Facts You Should Know about Security and Commodity Exchanges

B. 72—Facts You Should Know about Saving Money

B. 73—Facts You Should Know about Investment Companies

B. 74—Facts You Should Know about Investment Banking

B. 75—Facts You Should Know about the Consumer Education Study

B. 76—Facts You Should Know about Commercial Banks and Trust Companies

B. 77—Facts You Should Know about Budgeting

B. 78—Facts You Should Know about Buying or Building a House

National Better Business Bureau, Inc., Chrysler Building, New York 17, N. Y.

Booklets; $5\frac{1}{4}'' \times 8''$; Generally 8 pages each; $.05 each.

Description of B. 69 through B. 78: The titles above give the general nature of the pamphlets. They are well written and approach each problem from an elementary standpoint suitable for secondary mathematics classes. Typical problems are solved which illustrate the topics.

Appraisal of B. 69 through B. 78: These inexpensive, authoritative booklets should be used in all classes which study the consumer aspects of mathematics. Some junior high school classes will find the vocabulary a little advanced, but the teacher can use the booklet herself from which to draw information to pass on to the class. Sets should be available to loan to the pupils at all levels. The usefulness in schools would be greatly increased, however, if the material were made more attractive. There is a need for pictures, drawings and charts to enliven the appearance of the pages. Examples for pupils to do to follow up the reading would also be useful, but it is not too difficult to devise these—either the teacher or pupils can do so.

B. 79—Teaching Aids for Financial Security Education

B. 80—The Story of Life Insurance

B. 81—Careers for You in Life Insurance

B. 82—Life Insurance Fact Book

Institute of Life Insurance, 488 Madison Avenue, New York, N. Y.

Booklets; Free.

Description of B. 79: This is a general catalog which includes the other booklets mentioned above as well as other aids to teaching. There are 15 booklets on insurance and money management, 3 filmstrips, 4 wall charts, 2 comic books, and 5 sound films (16 mm.).

Description of B. 80: This unit on life insurance for the intermediate and upper grades is $8\frac{1}{2}'' \times 11\frac{1}{2}''$ and contains eight pages which are reprinted from the November, 1950, issue of *The Grade Teacher*. There is a short history of life insurance and a description of the present practices. Two pages of correlated activities are grouped under the headings of language arts, social studies, science and health, arithmetic, graphs and charts, arts and crafts, and cumulating activity. A short bibliography is included.

Description of B. 81: On 72 pages, $6'' \times 9''$, is a well-illustrated summary of the various positions in life insurance work. Among others are included the actuarial, medical, clerical, legal, and sales departments.

Description of B. 82: This fact book is filled with tables of figures concerning life insurance companies and their policies. It is an endless source of material for teachers and pupils to construct problems. The short mortality tables are especially useful to illustrate the facts upon which premiums are based.

Appraisal of B. 79 through B. 82: These are very useful aids for the sections of courses concerning insurance. They are so attractive that they will lead to excesses in stressing this topic.

The ability to find material on a subject in mathematics within a welter of similar information is a skill often neglected in mathematics classes. The usual problems in school contain enough information, but not too much. Is that reality?

CHARTS

C. 26—Automobile Stopping Distances

General Motors Corporation, Educational Service, Department of Public Relations, Detroit 2, Michigan.

Chart; $22'' \times 34''$; Black and white; Free

Description of C. 26: The front of the chart shows graphically the relative distances which a car moves during the driver's reaction time, and while stopping on dry pavement, wet pavement and ice. Each of these facts is shown for 20, 30, 40,

50, 60, 70 and 80 miles per hour. The back of the chart discusses factors which affect stopping time.

Appraisal of C 26: This is an excellent example of a statistical graph and always causes comments and interest in a class. From the obvious mathematical conclusion that the stopping distance is not a linear function of the speed, one could go on in more advanced classes to finding a formula which will fit the data given. In elementary classes the chart is useful to read graphs and compare lengths.

FILMS

F. 61—Areas

Knowledge Builders Classroom Films, 625 Madison Avenue, New York 22, N. Y. Educational Collaborations, R. C. Jurgensen.

16 mm. film; Black and white; Sound; 400 feet, 12 minutes; $40.

Description: The concept of area is illustrated by use of a square whose dimensions are one unit each. Several rectangles are presented and their areas found by counting the number of units of area. From these illustrations the formula for the area of a rectangle is assumed. The area formulas of the parallelogram, triangle, trapezoid, regular polygon and circle are then derived in order without formal proof. A parallelogram is compared to the rectangle by use of congruent right triangles, and it is stated that it can be proved that the area of a parallelogram also can be found by using the formula, $A = bh$. Altitude is illustrated and defined. A triangle is formed by the diagonal of a parallelogram and its area is shown to be half that of the parallelogram. Triangles having the same base and equal altitudes are shown to be equal in area by moving the vertex of the triangle so that it remains equidistant from the base. By dividing a trapezoid into two triangles and a regular hexagon into six congruent triangles two more formulas are derived. As the number of sides of the regular polygon is increased indefinitely, the apothem approaches the radius and the perimeter approaches the circumference of a circle. $2\pi r$ is substituted for circumference and r for apothem in $A = \frac{1}{2}ap$, and $A = \pi r^2$ is derived. A summary of the six figures, by use of diagram and formulas, concludes the film.

Appraisal: The concept of area is presented clearly without formal demonstration. Because a plan of proof is given for each of the six figures, this film must be shown at the beginning of the study of the unit on areas. Pupils then should be ready to study the formal proofs. All the plane geometry classes in our school saw the film, and all the teachers were pleased with the pupil reactions and results. Although the area formulas for the regular polygon and the circle are not studied at this time in our classes, their inclusion presented an interesting application of the triangle. The presentation and sound track are very clear. Every class in plane geometry can profit by seeing this film. The only objectional explanation in the film is the use of the word "cancellation" for divide in the derivation of the $A = \pi r^2$ formula. (Reviewed by Herbert Freed, Senior High School, Atlantic City, N. J.)

F. 62—Fixed Gauges

Producer: Division of Visual Aids for War Training, U. S. Office of Education, Washington 25, D. C.

Film obtainable from: United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

16 mm. film; Black and white; Sound; 18 minutes; 1 reel; $28.58; study guide available.

Description: This film shows in detail: (1) types and uses of snap gages; (2) types and uses of ring gages including taper ring gages and thread gages; (3) types and uses of plug gages including straight, taper, and screw plug gages; and (4) design and use of a flush pin gage.

The film opens by showing that every drawing contains the required information about dimensions, and how the allowable tolerances are stated. Many types of

fixed precision gages are shown including: snap gages, ring gages, plug gages, taper plug gages, taper ring gages, and their various uses.

The types and uses of snap gages are shown. These are fixed gages designed for taking outside measurement. The use of the go button and not-go button is illustrated. The correct way to gage a round piece is shown, also how to gage parallel sides. How adjustable limit gages can be set to a precise dimension is explained.

The types and uses of ring gages are then shown. This includes the not-go ring gage, the taper ring gage, the thread ring gage, the roll thread snap gage, and other special types. Demonstrations of their use is clearly brought out.

The types and uses of plug gages are shown. There are two principal classes, straight plug gages and taper plug gages. It is shown that the accuracy of any fixed gage is classified according to the type of work for which it is designed. That the flush pin gages are widely used for checking depth dimensions and that the screw plug gages are used for checking internal screw threads is illustrated.

A scene from the gage inspection or gage control department is shown where all gages and other measuring tools are checked and adjusted. A precision gage block is being used with an electrolimit gage to check the accuracy of a flat plug gage.

Appraisal: For the evaluation of this film there were seven teachers of mathematics present. They considered the entire film as good for the introduction of new material, to augment explanations, to provide a common experience, to motivate and for use in review. Four of the evaluators believed that the film was just fair for use in developing skills. It was agreed that the film could be certainly utilized from the 9th to the 12th grade, but only in shop mathematics. The entire group considered the speed of development of ideas and duration of scenes as being moderate and that there were no inaccuracies in the mathematical content. Four of the teachers saw the film as partially holding the interest of students while the remainder saw it as completely holding the interest. The teaching methods were partially conducive to five of the evaluators while the remainder found them to be completely so. The photography was good, and three of the previewers believed the sound to be good and three that it was fair. Four checked the effectiveness of characterizations and background objects as good, but there was one check each for very good, fair and poor. Five thought that the presentation would encourage further pupils activities in mathematics and in other subjects. Two of the previewers scored that the captions were not of appropriate length while all remarked that they were pertinent. The greater number considered the dialogue as effective. All agreed that there was coordination between sound and picture and that the arrangement of topics was in a logical sequence. The evaluators were four to three that the amount of material was reasonable for comprehension in one showing. Four felt that the content was not able to be just as effectively and efficiently presented in some other way. Five saw the film as supplementing rather than replacing the teacher. Finally, five of the teachers would definitely use the film. (Reviewed by A. diLuna, R. Fleet and K. Hathaway)

F. 63—The Steel Rule

Producer: Division of Visual Aids from War Training, U. S. Office of Education, Washington 25, D. C.

Film obtainable from: United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

16 mm. film; Black and white; Sound; 14 minutes; 1 reel; $25.71.

Description: Discusses in considerable detail the steel rule, emphasizing the variations of the steel rule, the type of scales found on them, their proper use, and the correct procedures in transferring

measurement by means of calipers and dividers.

Appraisal: Eight teachers were together for the evaluation of this film. Considering the entire film they found it as being good when used to introduce new material, to develop skills, or for review. Three of the teachers evaluated the picture as good for both augmenting explanations and providing a common experience, but one fourth also checked "fair" in these two purposes. When considering using the film for motivation, all categories were checked with three each in "good" and "fair." The grade level was placed between the 8th and 12th grade with marked predominance in the 9th. The film is especially useful in shop mathematics with some value in arithmetic. The speed of development of ideas is moderate and there are no inaccuracies in the mathematical content. Five previewers saw the film as partially interesting to pupils and three found it to be completely so. The same figures were also reported when they evaluated the teaching methods in the film as conducive to learning. Six scored the photography as good, but two scored it very good and 1 as poor. One half thought that the sound was good and the remainder were split on very good and fair. The effectiveness of characterizations and background objects was good to five and poor to one. The film presentation will encourage further pupil activity in mathematics as was penciled by three-fourths of the evaluators, and all agreed that it would in other subjects. All were in unison in saying that the captions were both of appropriate length and pertinent. The dialogue was definitely effective. The entire group voiced that the arrangement of topics was in a logical sequence and that there was coordination between sound and picture. Of the evaluators, five definitely believed that the content of the film could not be just as effectively and efficiently presented in some other way. Seven of the teachers did think that the film did attempt to supplement rather than replace the teacher. Finally, six would use the film if given the opportunity. (Reviewed by A. diLuna, R. Fleet and K. Hathaway)

F. 64—An Introduction to Vectors

Producer: Division of Visual Aids for War Training, U. S. Office of Education, Washington 25, D. C.

Film obtainable from: United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

16 mm. film; Black and white; Sound; one reel; 22 minutes; study guide available; $34.31.

Description: A number of key points are covered in this film such as: (1) explanation of the meaning of scalar and vector quantities; (2) explanation of how to add scalars and vectors; (3) explanation of the various methods of vector composition and vector resolution; (4) explanation of the relationship between vector composition and vector resolution; (5) clarification of the relationship between the analytical and graphical methods of vector composition and vector resolution; and (6) shows how vectors may be employed to solve engineering problems.

The film opens with a discussion of scalar and vector quantities. It is shown that the miles flown by an airplane may be represented by lines proportional in length to the number of miles covered. These quantities which have magnitude only are called scalar quantities and are added by simple arithmetic. Scalars have but one property, magnitude or amount. A plane is then shown flying from one city to another, and an explanation follows that in this instance two values are indicated, magnitude and direction. When quantities of this type have magnitude and direction, they are called vector quantities.

A graphic illustration of vector quantities is given. Line of action is defined and the fact that direction of the quantity indicates a vector.

The addition of vectors by the triangular method is shown. Two examples are given to illustrate this. In the first exam-

ple a plane is flying from Charleston to Pittsburgh, and it is shown how the combined action of the two displacements results in the plane's net displacement or the resultant. In the second example, a plane is flying east with a velocity of two hundred miles per hour and at the same time a wind is blowing from the north with a velocity of sixty-five miles per hour. The plane's displacement by vector addition is determined. These examples are illustrated by animation.

Another way is shown by which the resultant may be obtained. This is the parallelogram method. The last method shown is the analytical method. This method is discussed by use of a right triangle.

It is explained that vector quantities of any type can be combined by the same methods. An example given is a force of forty pounds acting in a certain direction applied to an object, and another force of thirty pounds acting in a different direction. The vectors in this problem are added first by the parallelogram method and then by the triangle method. It is shown that either method may be used, for the resultant is the same in both cases.

A discussion follows on the order in which the vectors are arranged in the triangle method and parallelogram method. In the triangle method of vector composition it is shown that the order in which the vectors are arranged, tail to head, makes no difference in the resultant, but the resultant always joins the tail of the first to the head of the second. In the parallelogram method the vectors are always joined together tail to tail, and regardless of their size or direction the resultant is always the vector drawn diagonally from the common point of origin.

What components are; how the pairs of components may be obtained; and the fact that the only components considered are those at right angles to each other is discussed. The plus and minus properties of the x and y components are illustrated by animation.

The x and y axes may be in positions other than the horizontal and vertical, but regardless of their position the method remains the same. This is shown in the example of a ball weighing one thousand pounds resting in a ninety degree angle plate, one side of which makes an angle of thirty degrees with the horizontal. The force which the ball exerts on each face of the block is determined by vector resolution which is carried out graphically.

It is shown that very often several quantities may act together. An example of four men pulling on a heavy object in certain directions may be represented by a vector, and the total effective pull on the object can then be determined. Both the triangle and parallelogram method are used to work this problem.

The film ends with a short review of the fundamental issues taken up.

Appraisal: Ten teachers of mathematics reviewed this film. Five thought the film was excellent for introducing new material while two thought it was poor for this purpose. Six said the film was good for developing skills. Five checked the film as excellent in providing a common experience and in reviewing. The grade level was placed between the 11th grade and the college level. 47% favored the 12th grade while 42% the college level. The film was considered useful in trigonometry and advanced mathematics. Two of the evaluators thought the speed of development of ideas and duration of scenes was too fast, while one found some inaccuracies in the film. 60% said the film would hold the interest of the students and that the teaching methods in the film were conducive to learning. 50% said the photography and sound were very good. All of the teachers said they would use this film in their classroom and the general comment was that the film represents an unusually effectively arranged presentation. (Reviewed by Anthony DiLuna, Raymond Fleet, Kenneth Hathaway)

FILMSTRIPS

FS. 90 to FS. 94—Introduction to Fractions

Jam Handy Company, 2900 East Grand Boulevard, Detroit, Michigan.

35 mm. Filmstrip; Black and white; Silent; $4.50 each, $19.50 per set.

Description: FS. 90, Part I, is entitled "The Fractional Parts of a Whole" and consists of 28 frames. The first four frames show "John and Susan" cutting an apple in half. The next frame shows a half-red, half-blue pencil. Then a circle on the blackboard with a diameter drawn in is shown. The division into thirds and quarters is also shown. Two frames are taken up showing one-third and one-fourth respectively written on a blackboard.

Description: FS. 91, Part II, is entitled "Fractional Parts of Groups" and consists of twenty-four frames. This filmstrip begins with "Johnny" having some marbles and then dividing them, first, into two parts, then into three parts, etc. Frame nine is concerned with counting or addition only. Several frames show only numbers such as $\frac{1}{3}$ of $12=4$, $3\overline{)12}^{\,4}$

Description: FS. 92, Part III, is entitled "Fractional Parts of a Whole and Groups" and consists of twenty-two frames. This filmstrip shows rabbit hutches, doll boxes, a container for a shell collection, and cupboards all divided into fractional parts. The students are then shown the number of things to be placed in the containers and asked what the desired fractional part will be. There are five frames in which only numbers are shown.

Description: FS. 93, Part IV, is entitled "Non-unit Fractions of the Whole and Groups" and consists of twenty-six frames. This filmstrip shows a garden divided into four parts, each fourth belonging to a different person. Then the question of how much of the garden three of the children own, two of the children own, etc., is asked. Again one frame is used in showing "$\frac{3}{4}$" written on the blackboard. The rest of the filmstrip shows such things as flowers and raspberries, and asks the children for the specific numbers of fractional parts of each example.

Description: FS. 94, Part V, is entitled "Comparing Fractions" and consists of twenty-five frames. The first five frames deal with halving candy bars, cutting a pie into thirds, etc. The sixth frame shows a kite with the four parts colored and asks whether each part is equal to one-quarter. From here on the filmstrip deals mostly with paper, different fractions of which are colored.

Appraisal of FS. 90 through FS. 94: The photography is excellent, the colors are attractive and the captions easy to read. Third and fourth grade children would probably enjoy the variety of colors and the change the filmstrips might provide, if they are accustomed to blackboard work entirely. However, there is little presented in the filmstrips that could not be shown in the classroom to better advantage. The numbers pictured by themselves could just as easily be shown on the classroom blackboard. (Reviewed by Dale A. DuVall, Mathematics Instructor, Boston University, General College.)

INSTRUMENTS

I. 33—Sighting Level

Scott and Anderson, Inc., 952 Dorchester Avenue, Boston 25, Mass.

Surveying instrument; 1″×1″×5″; $2.00 (FOB Boston).

Description: This level contains no lenses, but has a peep-hole at one end and a glass scale with cross-hair and scale at the other. A bubble level is fastened to the top and is reflected in a mirror so that it can be observed through the eye-piece at the same time that the instrument is sighted. By centering the bubble on the hair-line, one can hold the instrument horizontal. Combined with a measuring rod this level can be used to measure heights above and below the level of the observer.

Appraisal: It is remarkable how much mathematics can be derived from a simple level of this type, and how extraordinarily

accurate a simple, inexpensive device can be. Although home-made levels which the pupils build are effective in theory, this compact, professional-looking level is often more fun for them to use. Seldom is it justified to buy such equipment as this in quantity for a whole class. But if schools could only begin to buy single instruments of various types and thus build a collection of many different kinds, there would soon be a livelier interest and fuller understanding of the applications of mathematics.

I. 34—Slide Rule and Mathematical Tables

Standard Transformer Corp., 1500 N. Halsted Street, Chicago 22, Ill.

Cardboard slide rule and tables; $4'' \times 10''$; $.05 each.

Description: This rule consists of a cardboard case with cut-out sections to allow readings, a cardboard slide to fit in the case and a sheet of plastic which also slides within the case and has hairlines engraved upon it. One side of the device has *A*, *D* and *K* scales on the case and *B*, *CI* and *C* scales on the slide; the other side has *A* and *D* scales on the case and *S*, *L* and *T* scales on the slide. In addition there are formulas, decimal equivalents, and slide rule settings printed on the case. A unique feature is the sets of four-place logarithm tables, and natural trigonometric function tables (sine, tangent, cotangent, and cosine) which appear through a slot in the case as the slide is moved. The former require interpolation for the fourth figure of the number which enters the table; the latter is given by degrees from 0 to 90. A two-page set of directions is enclosed.

Appraisal: This ingenious and sturdy little device will be very useful in the classroom. There is a tendency for the slide to bind in the case, and this sometimes makes accurate settings difficult. Having only one side provided with a hairline restricts the usefulness somewhat. Also, the figures in the logarithm and trigonometric tables are extremely small and therefore somewhat hard to read. It should be noted, however, that the appearance of only one line in the table through the opening in the case is apt to decrease the possibility of reading the wrong entry from the table. This device is well worth adding to the mathematics collection for observation and for manipulation.

MODELS

M. 24—Computation Unit

M. H. Ahrendt, National Council of Teachers of Mathematics, 1201 Sixteenth St. N. W., Washington 6, D. C.

Science kit; $4\frac{1}{2}'' \times 6\frac{3}{4}''$; In envelope; $.50 each, 3 for $1.00

Description: This new kit of the mathematics series which is planned by the National Council of Teachers of Mathematics in cooperation with Science Service contains a booklet of instructions ($5'' \times 6''$, 12 pages), a sheet to cut out and mount of Napier's Rods ($8\frac{1}{2}'' \times 11''$), a sheet to cut out with addition-subtraction scales and slide-rule scales ($5\frac{1}{2}'' \times 12''$), and a plastic slide rule ($1'' \times 6\frac{1}{2}''$). The rule contains *A* and *D* scales on the base and *B*, *CI* and *C* scales on the slide. When the slide is withdrawn it discloses inch and metric scales on the edges. The booklet describes 28 "experiments": 6 on the addition-subtration rule, 7 on the slide-rule, 4 on logarithms, 5 on square root, and 6 on the Napier's Rods.

Appraisal: Since small plastic slide rules are so expensive this kit is probably marketed at a fair price. The small rule is useful, however, only to learn the theory of the slide-rule, since it is so short and the accuracy of the scale is to be doubted. The idea of cutting out scales and mounting them on cardboard is excellent. Certainly every school should have one of these kits; from the paper cut-outs the pupils can make similar ones, then the school can purchase better slide rules to add to the collection.

(*Continued on page 517*)

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
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THE PROBLEM OF PURSUIT

AMONG THE problems of frequent occurrence in textbooks in arithmetic and algebra are those which deal with the length of time needed for one traveler to overtake another when the velocity of each is known and when the difference in the starting time of each is given. It is possible that these had their origin in the use of pace-scales in the ancient world but this is a matter of conjecture. It is common observation, however, that if a person walks at a uniform rate for a given time, the distance can readily be computed. For the purpose of making a problem, uniform rates were assigned in somewhat unlikely situations and time intervals were stated even when the author of the problem was unable to determine the interval with the desired precision. For examples of this, consider the problem of the hare and the hound. The first of these examples is from the collection attributed to Alcuin (*c.* 775) and the second is from the work of the Italian Cardan (1539).

> A field is 150 ft. long. A dog stands at one corner and a hare at the other. The hound goes nine feet in each leap, the hare seven. In how many feet and in how many leaps will the hound catch the hare?

> A dog is chasing a hare. The hare is 60 leaps ahead of the dog and three leaps of the dog equal five of those of the hare. The dog takes three leaps in 20 seconds and the hare 5 in 21 seconds. When does the dog catch the hare?

The problem of pursuit where one man catches up with another permits the use of a large amount of human interest material as is seen in the following examples:

> Reuben sets out from his city in the morning of the first day of the new moon to go to meet his brother Simon in Simon's town. On the selfsame day, Simon also leaves his town to go to see Reuben in his city. The distance between the two places is 100 miles. Reuben goes 19 miles a day. Simon goes 17 miles a day. We ask when they will meet.—Rabbi ben Ezra (*c.* 1140)

> Two citizens of Oppenheym called Heynrichson and Contz von Treber, decide to go to Rome together. Heynrichson was old and could travel no more than ten miles a day. Contz was young and strong and could go 13 miles a day. Accordingly, Heynrichson started out from Oppenheym nine days before Contz left, so he was 90 miles away when Contz started. The question is, in how many days will Contz overtake Heynrichson?—Köbel (1514)

> A certain slave fled from Milan to Naples going 1/10 of the whole journey each day. At the beginning of the third day his master sent a slave after him and this slave went 1/7 of the whole journey each day. I do not know how far it is from Milan to Naples, but I wish to know when the second slave overtook the first.—Cardan (1539)

> Suppose *A* leaves Newburyport at 6 o'clock on Monday morning, and travels toward Providence at the rate of 4 miles per hour without intermission: and that at 3 in the afternoon *B* sets out from Providence for Newburyport and travels constantly at the rate of 7 miles per hour; Now, suppose the distance between the two towns to be 90 miles; whereabout on the road will they meet?—Nicolas Pike (1788)

> A man who had stolen a horse rode away on its back. When he had gone 37 Chinese miles, the owner discovered the theft and pursued the thief for 145 miles. He then turned back being unable to overtake him. When he turned back the thief was riding 23 miles ahead of him; if he had continued in his pursuit without coming back, in how many miles would he have overtaken him?—Chang Ch'iu-chien (*c.* 575)

The variants of the problem include all sorts of conveyances. The speed may vary as is shown in the following example which occurred in several works of the fifteenth and sixteenth century in Italy.

> There are two birds in a tree. One flies toward the east going 1 mile the first day, 2 the second, 3 the third and so on. The second flies toward the west going 1 mile the first day, 8 the second, 27 the third and so on according to the cubes of the numbers. The distance around the earth is believed to be 44,310 miles. In how many days did the two birds meet?—Cardan (1539)

From this situation, the problem took on an astronomical application.

> If Saturn revolves around the earth in 30 years and if Jupiter goes around it in 12 years, how many years will there be in the interval between conjugations?—Cardan (1539)

The situation was applied also to the question of when the hands of a clock are together. A Chinese problem has a simpler case where the problem is not complicated by astronomical terminology—

> There are three sisters of whom the eldest comes home once in every five days, the middle one once in every four days, and the youngest one once in every three days. In how many days will all three meet together?

The problem of the couriers, the problem of pursuit, the problem of the planets, whatever it is called, keeps reappearing in many different forms, but when a class has been working with one of these as an exercise in the analysis of a problem situation, introduce the Chinese problem as given above and then follow it with this problem which is taken from W. W. Sawyer's *Mathematician's Delight* published by the Penguin Books Ltd., London, 1943.

> Albert Smith and his wife Betty are both in the forces. Albert is off duty every ninth evening; His wife is off duty every sixth evening. Albert is off duty this evening: Betty is off duty tomorrow evening. When (if ever) will they be off duty the same evening?

Aids to Teaching

(*Continued from page 515*)

RESOURCES

R. 2—Our Banks and What They Mean to Us

Dr. Kenneth P. Kidd, College of Education, University of Florida, Gainesville, Fla.

Tentative Resource Unit; 8½″×11″, 47 pages, mimeographed; $.35.

Description: Twenty-two pages of this booklet are devoted to background material in the form of text which explains the organization and services of a bank and how to use them. Then there are five pages of a glossary of terms and the remainder is a source of teaching suggestions. The teaching suggestions list the goals of the unit, materials (books, films, periodical articles, and pamphlets and posters), a discussion of dramatization, bank tours, and the use of displays and posters, and other miscellaneous suggestions.

Appraisal: Since this unit is in a tentative form there are places which merely state that a certain form will appear there in a later edition; this is easily supplemented by local illustrations, but it gives the booklet an unfinished appearance. It is unfortunate that so many references must be made to a local source, but then the Florida references will suggest similar addresses in other communities. The arrangement of the material, the type of material chosen and the text explanation are quite satisfactory and should suggest to other groups how to build similar units. It is good to have samples of this kind as examples of the meaning of "resource unit."

It has been called to our attention by the National Forum, Inc., that our review of their social studies charts (C. 20) should have said that the charts can now be purchased by individual selection; 5 to 9 charts—$.60 each, 10 to 29—$.55, and 30 or more charts—$.50. We are glad to make this correction.

RESEARCH IN MATHEMATICS EDUCATION

Edited by JOHN J. KINSELLA
School of Education, New York University, New York, N. Y.

The Question: What should constitute the program of tenth year mathematics?

The Study: Hlavaty, Julius H. *Changing Philosophy and Content in Tenth Year Mathematics.* Teachers College, Columbia University. 1950.

FOR SOME time there have been serious doubts that plane demonstrative geometry constitutes the best mathematical education for the tenth year. Dr. Hlavaty sought "to examine the situation that exists in the teaching of demonstrative geometry, to investigate the influences that have brought it about, and to explore the directions in which there seem to be promising answers to the problems raised by it." (P. 2.)

An examination of the history of geometry in English secondary education was found necessary because early American educational history is a story of transmission of ideas from the Continent. In analyzing trends in this country the investigator concentrated on three periods, namely, 1890–1923, 1923–1939 and 1939–1948.

To study "the situation that exists" Dr. Hlavaty supplemented his analysis of significant reports and addresses by a searching diagnosis of eighteen American geometry textbooks published in recent years.

"To explore the directions in which there seem to be promising answers" the investigator integrated his previous findings with the recommendations and practices of outstanding leaders in mathematical education and general education.

Findings

The English experience revealed over the years a decrease in the emphasis on deduction and an accompanying increase in the use of experimentation and inductive procedures, a reduction in memorization of proofs, the acceptance of hypothetical constructions, the grouping of propositions into topics or units, greater attention to generalizations of results, and more use of the analytic method of proof. In more recent years English schools have been having difficulty in deciding the place of demonstrative geometry in an education for all adolescents.

During the period, 1890–1923, in the United States "originals" were not used much while verbatim recitation of proofs was still the accepted practice of the majority of teachers. This period was terminated by the appearance of the famous 1923 report[1] which concluded that geometric "facts" should be acquired in the junior high school, that demonstrative geometry should be an elective and that the aims of geometry should include the development of spatial imagination, the mastery of the great propositions and their applications, the understanding of deductive proof, the acquisition of habits of precise statement and logical organization of ideas and the growth of logical memory. It was apparent that the automatic transfer of mental characteristics was still considered to be within the realm of possibility.

During the period 1923–1939 there was a decline in the percentage of high school students electing geometry but an increase in the number. The Third Report of the Committee on Geometry[2] recommended in 1935 that transfer materials of a non-mathematical sort be provided.

In the 1939–1948 interval, the Joint

Commission Report of the National Council of Teachers of Mathematics and The Mathematical Association of America[3] as well as the Progressive Education Association's "Mathematics in General Education"[4] were published. The former recommended only minor curricular modifications for the tenth year while the latter, in its chapter on proof, reflected Fawcett's thinking[5] that geometry must be so taught that improvement in reflective thinking in non-mathematical situations would also be achieved.

For the period, 1930–1948, Dr. Hlavaty discovered an emphasis on practical geometry, attention to the nature of proof in non-mathematical situations in addition to geometric ones, a radical break with the Euclidean approach in the work of Birkhoff and Beatley,[6] Swenson's advocacy of integrated mathematics,[7] and the coming of the general mathematics movement. He also found that geometry was more frequently being considered "as an elementary illustration of the scientific method by encouraging experimentation and clarifying the complementary roles of deduction and experiment" (p. 91), an emphasis on three dimensions as well as two, a tendency to use instruments other than the straight-edge and compass and a modification of the sequence of topics "to permit a more gradual approach to formal proof." (P. 91.)

His analysis of geometry textbooks from 1930–1948 revealed, among other trends, a decrease in the number of propositions, an increase in the number of assumptions, originals and practical applications, more attention to reasoning in non-mathematical situations and the appearance of materials from analytic geometry.

Finally, Dr. Hlavaty proposed for the tenth year a program. "The method traditionally associated with demonstrative geometry is a major objective of the course but it is to be developed with arithmetical, algebraic, non-mathematical as well as geometric materials." (P. 112.) Considerable emphasis would be placed on applications. Deduction would be only one of the aspects of thinking considered. The experimental-inductive procedures of scientific thinking would be examined along with probability and statistics. Of the eight units proposed for the course only one would be devoted entirely to traditional plane geometry; only one or two sequences, e.g., congruence, areas, circles, would be treated in this unit. The remaining units would involve coordinate geometry, trigonometry with simple proofs, geometry of the global earth, statistics and probability, thinking in non-mathematical situations and arithmetic and algebraic proofs and rationalizations.

It is quite apparent that such a course would be of value to a much greater percentage of tenth year students than demonstrative geometry alone.

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6. Birkhoff, G. D. and R. Beatley, "A New Approach to Elementary Geometry," *Fifth Yearbook of the National Council of Teachers of Mathematics. The Teaching of Geometry*, pp. 86–95 (1930).
7. See for example "Integrated Mathematics," (John Swenson). *Secondary Education*, vol. 7 (April 1938), pp. 82–85.

The Association of Teachers of Mathematics in New England
Fourth Annual Institute for Teachers of Mathematics
August 21-27, 1952, The Phillips Exeter Academy, Exeter, N. H.

DEVICES FOR A MATHEMATICS LABORATORY

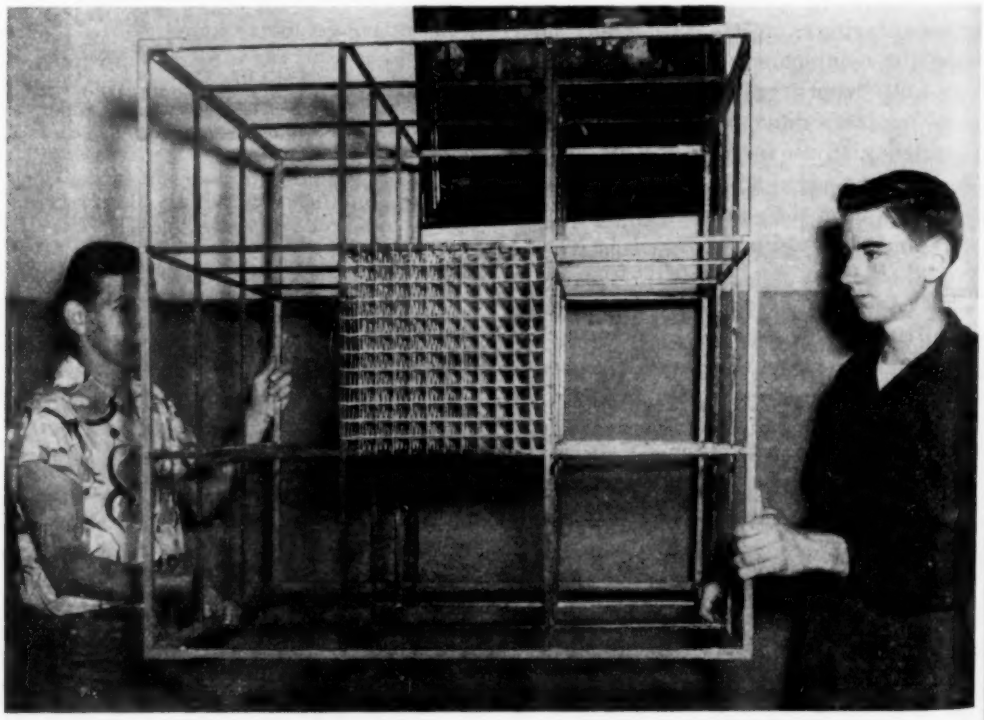
Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are requested to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning **Devices for a Mathematics Laboratory** to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

THE SUBDIVIDED CUBIC YARD—A STUDENT'S ACCOUNT OF ITS CONSTRUCTION

Few students have adequate conceptions of the actual size of the cubic units, to say nothing of the square units and linear units. Without this knowledge, problems involving the use of such units are devoid of meaning.

The model illustrated in the accompanying picture shows simultaneously the inch, the square inch, the cubic inch, the foot, the square foot, the cubic foot, the yard, the square yard, and the cubic yard. By examining the model students can gain a clear idea of both the actual and relative sizes of the common units of mensuration. Also they will be able to tell by inspection or simple calculation how many smaller units are contained in each



Pictured holding the subdivided cubic yard which they produced are Walter Schoenfeld and James Gavin, students at Cleveland High School, St. Louis, Missouri.

larger unit. The model in the picture was produced by the two students shown holding it; their names are James Gavin and Walter Schoenfeld. The two boys constructed the subdivided cubic yard in connection with their work in solid geometry at Cleveland High School, St. Louis, Missouri. The following résumé, written by James Gavin, is an actual account of how the boys constructed the model.

Construction of the Subdivided Cubic Yard

Our work on the construction of the subdivided cubic yard began in my garage on the evening of March 2, 1950. For the framework of the model we decided to use scrap poplar wood. Using a power saw, we ripped out twelve pieces $\frac{1}{2}'' \times \frac{3}{4}'' \times 37''$ for the framework of the cubic yard. The extra inch was allowed for later trimming so as to have accurate dimensions. We also sawed out 108 feet of sticks $\frac{3}{8}''$ square for the framework of the cubic feet. To make the cubic inches we bought 450 feet of chalk-line and dyed it yellow to make it show up better.

In order to make the model rigid we made eight corner braces from sheet aluminum. A model of the brackets was first made from paper. Then the aluminum brackets were cut according to this paper pattern. (See Fig. 1.) Using $\frac{1}{8}''$ stove bolts $\frac{1}{4}''$ long, we assembled the framework of the cubic yard with little difficulty.

Next we proceeded to construct the 27 cubic feet. First step in the process was to complete the framework of the faces of the cubic yard. For building material we used some of the $\frac{3}{8}''$ square sticks which we had previously cut. Again we cut the sticks slightly longer than three feet so they could be trimmed to the proper length when fitted into the framework of the cubic yard. Also we cut notches $\frac{3}{8}''$ wide and half-way through on each of these sticks at one-foot intervals. This enabled us to have flat surfaces at the places where the sticks crossed each other. The framework for the 54 square feet of surface area was assembled with fish glue and wire

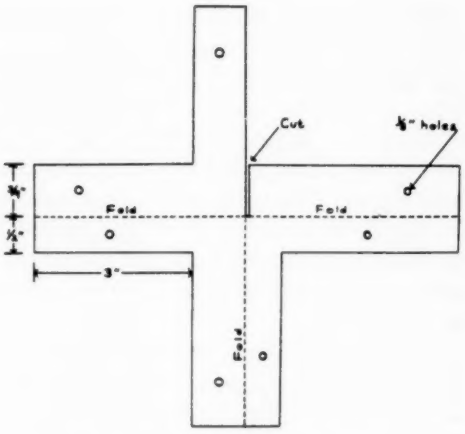


FIG. 1. Diagram of corner braces.

brads. To further strengthen the model, we made 12 "L" shaped aluminum brackets to fasten the newly added sticks to the edges of the cubic yard wherever they came together. In constructing the inside framework of the cubic yard we first formed two parallel planes with eight 37″ sticks—four for each plane. Then we fitted 12 one foot sticks into place, fastened them with brads and glue, and the outline of the 27 cubic feet was complete.

The most difficult part of our project was the construction of the 1728 cubic inches in one of the cubic feet. We considered using wire for this job but finally decided to use string. The strings dividing the surface of the cubic foot into 864 square inches were put in place first. A paper stapler was used to fasten the strings to the wood, and a dab of aeroplane cement was daubed on the vertices of all the little surface squares in order to keep their shape intact while the inner strings were put in place. Then, with the aid of a long needle fashioned from wire, we wove foot long strings between corresponding intersections on opposite sides of the cubic foot. In order to join the ends of loose strings to those that were already taut, aeroplane cement was again used to eliminate the need for tying knots. This part of the project took the longest time. After finishing the weaving we sprayed the net-

work of string with shellac to prevent it from shrinking and also to stiffen it.

Finally we painted the model orange and blue, our school colors. The cubic yard was painted orange, and the cubic feet framework, blue. The network of string forming the 1728 cubic inches, as previously stated, was dyed yellow. On April 27, the model was finished. A total of approximately 72 hours had been spent in making it.

E. M. Schueneman
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A Triangle Area Device

The area device illustrated in Fig. 2 was developed by a tenth grade girl in plane geometry class. The reader will enjoy the unique little domestic touches which are part of its construction features.

To build the device first procure a piece of board 8″×14″. Sand one side of it, rule it into ½″ squares, and apply a coat of shellac so that the lines will not smear when the device is used.

Next dig the family button box out of the sewing basket and select five buttons—four small white pearl tailor buttons and one metal shank button. Obtain a thin stout piece of wire 14″ long and about 16″ of colored elastic. Pass both the wire and the elastic through the hole in the back side of the shank button, and bend each end of the wire into a small loop. The materials are now ready to be assembled.

Place the iron wire on the board so that it will coincide with the top horizontal ruled line. Slip a tailor button under each of the two wire loops, and secure both the wire and the buttons to the board with small household nails. The two tailor buttons must be between the board and the wire so that the metal shank button will have clearance when it is moved along the wire. Nail the two remaining tailor buttons loosely to the board on the same horizontal line near the bottom edge of the board. They should be about 6″ apart. Finally tie the two loose ends of the elastic together and slip the completed loop around and beneath the two bottom buttons.

By sliding the shank button along the iron wire any number of triangles with a common base and equal altitudes can be outlined with the elastic. The area of any one of them can be found approximately in terms of the square units marked off on the board by counting the number of squares

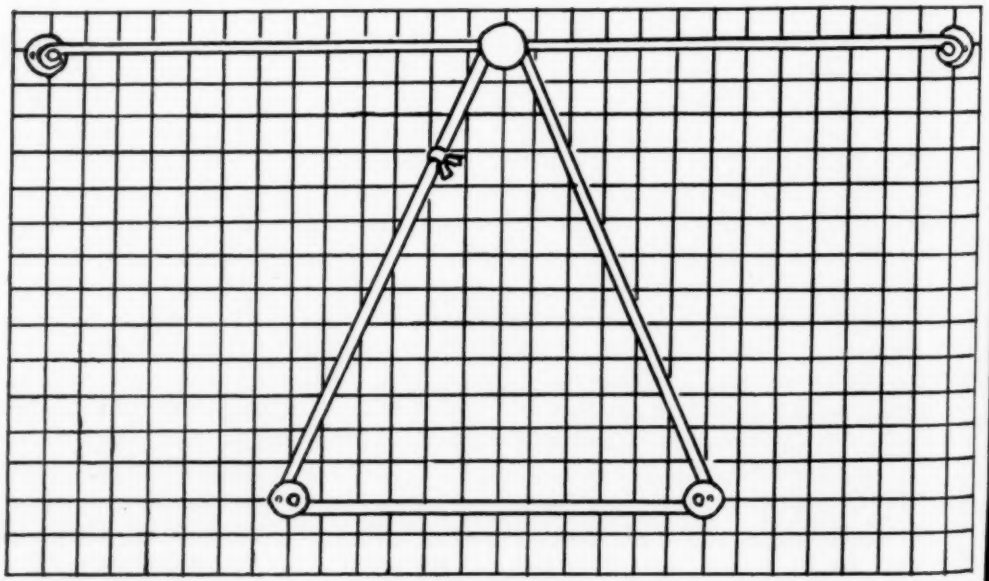

Fig. 2

included within its sides. When determining area in this manner every square should be counted as one full unit of area if more than half of it lies within the boundaries of the triangle. If more than half of a square lies outside the triangle it should not be counted at all.

By using this device students can discover informally that all triangles which can be formed with the device are equal, or what is the same thing, that all triangles with the same base and equal altitudes are equal. The device is of value because it provides a means by which students can arrive at a generalization. If the device is used in junior high school general mathematics classes this is usually sufficient. In the plane geometry class the device can be used to help students formulate the statement of the generalization which may then be proved deductively.

Another generalization which can be arrived at in an analogous manner through the use of the device may be stated as follows: The locus of the vertices of all the equal triangles that can be constructed on a common base is a line parallel to the base and at the distance of the altitude of any one of the triangles from the base.

THE MATHEMATICS LABORATORY
Monroe High School

GRAPHING DEVICE

The device suggested here can be used in both elementary and second course algebra classes to help give meaning to linear equations in two variables. It's amazingly simple to produce—so simple in fact that every student in class can with a little direction devise one of his own. The description given in this article is based on a device produced by an eleventh grade second course algebra student.

Materials needed include one piece of plywood $\frac{3}{8}'' \times 15'' \times 15''$, two pieces of welding rod 15″ long, two wooden beads which are large enough to slide over the welding rods, four small blocks of soft wood, four metal staples, one piece of elastic 15″ long, twenty-six round key tags, and two thumb tacks.

To build the device first mark the plywood board into $\frac{1}{2}''$ squares with a heavy black pencil. Make the center vertical and horizontal lines more prominent than the others. If a really finished product is desired the plywood should first be painted white, then marked into squares, and the completed grid coated with shellac. Next fasten the ends of the elastic to the wooden beads; the elastic may be passed through the bead holes and knotted securely on the outside as is shown in the diagram. (See Fig. 3.) Place one of the little soft wooden blocks at each corner of the board, slip one bead over each of the welding rods and staple the rods down over the blocks along the vertical edges of the board. One admonition should be followed with respect to selecting the blocks; they must be thick enough so that the beads can slide freely along the welding rods without scraping against the board. To make the device useful with as many different equations as possible letter each of the twenty-six key tags with a different letter of the alphabet.

The device described above is probably best suited for use by individual students. A workable demonstration device can be constructed by increasing the dimensions proportionally and substituting curtain rods for the welding rods. For this larger device ordinary metal curtain rod wall brackets will work out much better in fastening the rods to the board than the wooden block and staple arrangement used with the welding rods. If the beads selected for the smaller device are inadequate for the demonstration device, Tinker Toy wheels with the center holes drilled larger will work out quite nicely as a substitute.

In elementary algebra classes the device can be used in a variety of situations involving relations between two variables. As soon as students have learned how to plot points, and are satisfied that two points determine a straight line, experimentation with a sequence of equations

like those listed below should help them understand the significance of the constant in those equations.

$$y = x$$
$$y = 2x$$
$$y = 4x$$
$$y = -2x$$

How much meaning students eventually attach to situations of that kind will of course depend on the skill of the teacher. By thumb-tacking appropriate letters on the two axes the teacher can assist students in making real transfer to practical situations. Consider the following:

$$C = \frac{22}{7} d$$

$$C = 2\left(\frac{22}{7}\right) r$$

$$I = .05t$$

$$P = 4s.$$

Once students discover the scheme it will be a challenge to their insight to ask them to write down relations for two variables for different positions of the elastic line.

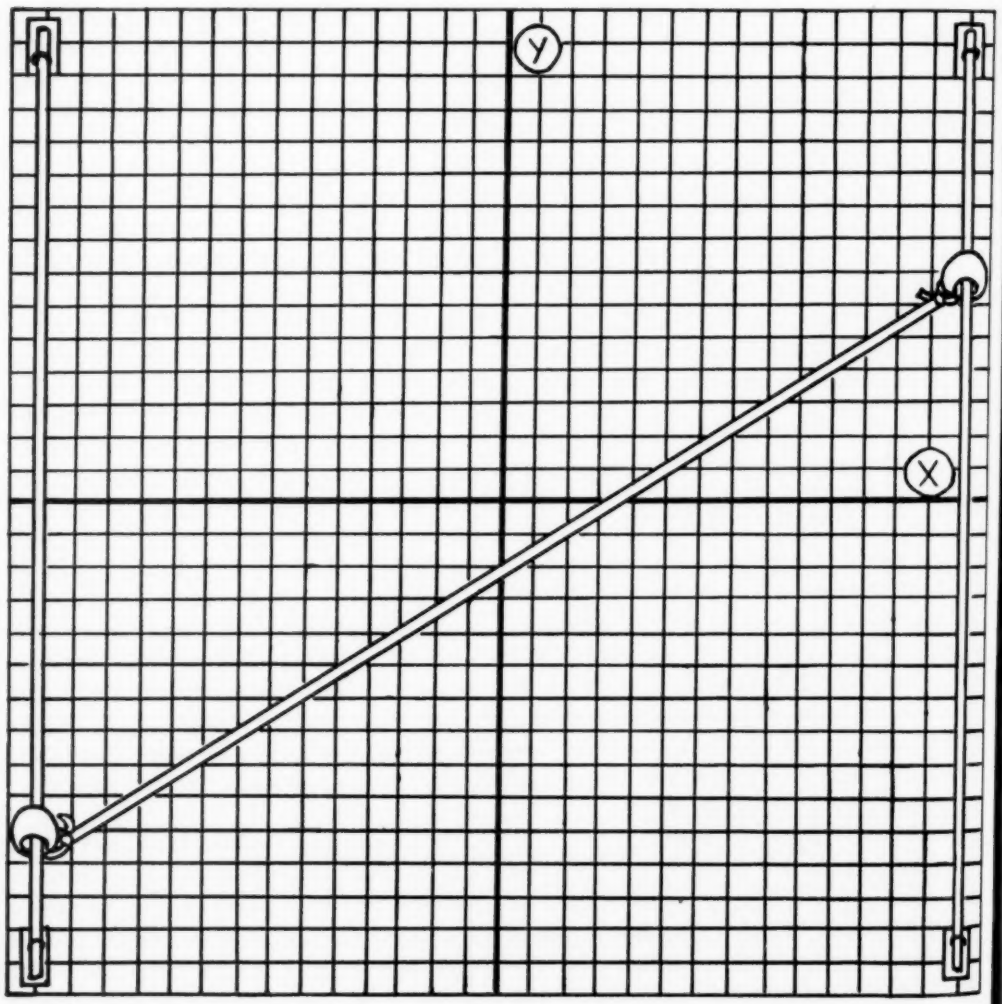

FIG. 3

Another suggested sequence of equations that should prove fruitful in terms of pupil understanding is the following:

$$x+y=5$$

$$2x+2y=8$$

$$2L+2W=6$$

$5n+p=10$ (nickels and pennies relation).

Consider now some of the possibilities that might be tried in a second course algebra class. The notion of what constitutes a solution of an equation in two variables can be made meaningful by experience with a variety of different lines. Such concepts as slope, intercept, and slope-intercept form can be made meaningful by a rapid consideration of a large variety of lines in different positions. In connection with the last named concept a nice application can be developed by considering the following relations:

$$y=9/5x+32$$

$$F=9/5C+32.$$

In order to handle problems which involve pairs of simultaneous, inconsistent, and dependent equations a second elastic line is needed. A convenient way of holding this second line in place is by using map pins or plastic handled bulletin board pins.

The illustrations given thus far are only suggestive of the uses which can be made of this device. Lack of space precludes further enumeration; however, a teacher who has the device in hand will undoubtedly be able to work out many uses by himself.

THE MATHEMATICS LABORATORY
Monroe High School

COMMENT ON "AN ELLIPTICAL BILLIARD TABLE"

Some years ago, I built a device equivalent to that described on page 405 of the December, 1950 issue of THE MATHEMATICS TEACHER. I found that while it was fairly satisfactory it seemed to have some shortcomings. The main difficulty seemed to me to be associated with imperfect rebounding.

The "Elliptical Whispering Gallery" depends not only on the tangent property of the ellipse which insures that the sound is reflected from one focus to the other, but also on the well known constant sum of focal radii because of which the sound reaches the other focus by all paths simultaneously. The device mentioned does not seem to illustrate this second property except in connection with its manufacture.

A contrivance which I have found to be in some ways more satisfactory is constructed in an essentially similar manner except that the base is of transparent plastic and the joints are carefully sealed. This is then mounted as the top of a lightproof box with a small source of light at the center of the bottom of the box. If an inch or so of water is placed in the elliptical top, shadows of ripples on the surface of the water may be projected upon the ceiling of a darkened room.

If, after the water has become still, a pebble is dropped into it at one focus, clearly defined circular ripples spread from it and after reflection converge circularly to the other focus.

FRANK HAWTHORNE
Hofstra College
Hempstead, New York

Warning of "The grave shortage of scientists and engineers which we face in this country at the present time," Dr. James R. Killian, Jr., President of the Massachusetts Institute of Technology, stated that this shortage will become still more acute in the next few years. "Last year," Dr. Killian said, "the engineering schools of the country graduated about 50,000 young engineers. This, of course, was an exceptionally high number, reflecting the wave of G.I.'s which had been going through the colleges. Based upon present enrollments, estimates have been made which indicate that in 1954 we will turn out of our engineering schools from 12,000 to 16,000 engineering students. This compares to an output from our engineering schools just before World War II of some 20,000 engineers a year.

REFERENCES FOR MATHEMATICS TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

Mathematical Plays and Programs

IN THESE days of group activities and similar undertakings, teachers in search of material appropriate for their particular needs may find the following notes helpful.

A. PLAYS, SKITS AND PAGEANTS

"A V Mathematics Playlet." MATHEMATICS TEACHER, 1945, 38: 309–313.

A costume play depicting the use of mathematics in business, science, music, art, etc.

Anning, Norman. "Socrates Teaches Mathematics." *School Science and Mathematics*, 1923, 23: 581–584.

A short play in one act; depicts the expansion of $(x+y)$ and $(x-y)$ by the use of blocks.

Brownell, Ella. "Mathesis." MATHEMATICS TEACHER, 1927, 20: 459–465.

Costumes and properties simple; incidental music; fifteen or more characters. Appropriate for junior high school.

Cohen, Dena. "Problem Play: a Musical Play in One Act and Three Scenes." MATHEMATICS TEACHER, 1936, 29: 78–83.

In which King Math of Numberland helps Alice to solve a problem in fractions; for junior high school. Detailed directions for costumes and scenery.

Cowley, Elizabeth. "A Mathematical Fantasy." *School Science and Mathematics*, 1933, 33: 535–536.

Brief skit (10 minutes) showing that numbers are indispensable in the modern world; fourteen players, no properties.

Crawford, Alma E. "A Little Journey to the Land of Mathematics." MATHEMATICS TEACHER, 1924, 17: 336–342.

A play in which two high school pupils get a vision of the value of mathematics in chemistry, drafting, bookkeeping, etc.; calls for twenty-six characters; suitable for ninth grade or junior high school.

Denbow, Carl. "Traders and Trappers." MATHEMATICS TEACHER, 1941, 34: 61–65.

A short play dealing with the principle of place value and the number concept in arithmetic; suitable for junior high school.

Fletcher, R. "A Quarter After Ten." *Texas Outlook*, 1931, 15: 31–35.

A mathematical play in one act.

Forrest, Louise. "Euclid, Agrarian Arbitrator." MATHEMATICS TEACHER, 1932, 25: 22–26.

Sketch based upon the concept of congruent triangles; requires four main characters, simple scenic properties. Time, about twenty minutes; for tenth year geometry classes.

Fort, Tomlinson. "Mechanics—A Dramatic Skit." *American Mathematical Monthly*, 1931, 38: 42–46.

Sophisticated skit depicting the relationships between mathematics and modern physical science; nine characters; no properties. For college students.

Gowan, J. C. "Mathematics Skit." MATHEMATICS TEACHER, 1943, 36: 169–170.

A very short play showing how mathematics is used in everyday life.

Graff, Margaret. "Euclid Dramatized." *School Science and Mathematics*, 1921, 21: 381–382.

A brief dramatization of some of the stories of early mathematicians.

Hall, Alton L. *The New Assistant.* The Metric Association, 1938. (10¢+postage.)

Deals with the spread of the metric system throughout the world; about fifteen characters, ten speaking parts. Pamphlet may be obtained from J. T. Johnson, 177 N. Grove Ave., Oak Park, Ill.

Harding, P. H., *et al.* "A Mathematical Victory." *School Science and Mathematics*, 1917, 17: 475–482.

A play in two acts for more mature students.

Hatton, Caroline and Smith, Doris. "Falling in Love with Plain Geometry." MATHEMATICS TEACHER, 1927, 20: 389–402.

A comedy; list of properties and directions for dances and musical numbers; sixteen characters. Excellent for senior high school.

Karapetoff, Vladimir. "Agha and Math." *Scripta Mathematics*, 1946, 12: 153–159.

A fantasy on "the way the logarithms might have been discovered even though they weren't." Readily adapted for dialogue, with a narrator. For high school and college.

Leps, J. M. "The Radical Dream: A Playlet for Puppets." *School Science and Mathematics*, 1933, 33: 279–287.

A play in two acts for algebra students; eleven characters.

Leskow, Olive. "The Cubic." MATHEMATICS TEACHER, 1943, 36: 312–316.

Depicts the contributions of Cardan, Tartaglia and Ferrari; eight characters.

Lloyd, Daniel. "In Quest of Truth." *School Science and Mathematics*, 1951, 51: 275–285.

A historical pageant, suitable for high school, college, or adult audiences. In two parts; playing time for each part, 20 minutes. Part I, suitable for junior high school, can be given independently.

McSorley, Kathryn. "Mock Trial of B versus A." *School Science and Mathematics*, 1918, 18: 611–621.

A clever dramatization based upon Stephen Leacock's celebrated story "A, B, C."

Meals, Louise. "X on the Spot." MATHEMATICS TEACHER, 1939, 32: 296–300.

A short play on the practical values of learning mathematics; six characters, minimum properties. Junior or senior high school.

Millay, Kathleen, and Free, Lucile. "Flatlanders." *American Mathematical Monthly*, 1919, 26: 264–267.

A brief play for four players; simple properties. Based on Abbott's "Flatland." See also *School Science and Mathematics*, 1914, 14: 583–587, as well as *The Mathematical Gazette*, 1914, 7: 228–231.

Miller, Florence B. "An Idea That Paid." MATHEMATICS TEACHER, 1932, 25: 470–479.

A play in three acts dealing with the subjects of profit and loss, partnerships, corporations, and stocks.

Miller, Florence B. "A Near Tragedy." MATHEMATICS TEACHER, 1929, 22: 472–481.

A play in three scenes showing the effects of having no mathematics; suitable for junior high school.

Miller, Florence B. "Out of the Past." MATHEMATICS TEACHER, 1937, 30: 366–370.

A two-act play written by a ninth-grade class, dealing with the development of our number system and the use of mechanical devices for computation.

Novinger, F. F., *et al.* "I had the Craziest Dream." MATHEMATICS TEACHER, 1947, 40: 20–23.

A play in three scenes dealing with practical applications of mathematics; fifteen characters, for junior high school.

Parkyn, Hannah. "When Wishes Come True." MATHEMATICS TEACHER, 1939, 32: 16–24.

A play about algebra and geometry; thirty-three players; detailed directions for costumes; for senior high schools.

Paterson, Edith. "Everyman's Visit to the Land of the Mathematicians." MATHEMATICS TEACHER, 1938, 31: 7–18.

Fairly elaborate in scope, dealing with mathematics from ancient to modern times. Includes notes on scenery, properties and costumes.

Pitcher, Wilimina. "Alice in Dozenland." MATHEMATICS TEACHER, 1934, 27: 390–396.

A play suggesting the nature of the duodecimal system; three scenes, five characters, exceedingly simple properties.

Pitcher, Wilimina. "The Mathematics Club Meets." MATHEMATICS TEACHER, 1931, 24: 197–207.

A historical play on methods of computation among the Egyptians, Romans, Italians; Russian peasant method of multiplication; sixteen players; modest settings and costumes; for senior high schools.

Potwine, E. B. "Bobby Learns His Numerals." MATHEMATICS TEACHER, 1943, 36: 29–33.

Depicts the history of numbers; eleven characters; presentation time, twenty minutes; calls for preparation of fifteen fairly elaborate charts. Suitable for junior/senior high school.

Raftery, Gerald. "The Eternal Triangle." MATHEMATICS TEACHER, 1933, 26: 85–92.

Simple playlet dealing with indirect measurement; three characters. Suitable for junior high school or ninth grade.

Reid, Noma. "Archimedes: A Mathematical Genius." *School Science and Mathematics*, 1941, 41: 211–219.

Outline of an interesting costume play describing some of the contributions of Archimedes to science and mathematics. Lines to be supplied by the players.

Russell, Celia. "More Than One Mystery." MATHEMATICS TEACHER, 1933, 26: 477–481.

Light skit, based on puzzle problems; seven players, virtually no properties; songs and dances. Appropriate for junior high school.

Schlauch, Helen. "Point-College." *School Science and Mathematics*, 1931, 31: 448–454.

A collegiate mathematical farce concerning Analytics College, in which the student body is composed of animated points; fifteen players.

Schlierholz, Tillie. "A Number Play in Three Acts." MATHEMATICS TEACHER, 1924, 17: 154–169.

A dramatization of Prof. D. E. Smith's *Number Stories of Long Ago*. Suitable for junior high school.

Scott, Emma. "Geometry Humanized." MATHEMATICS TEACHER, 1928, 21: 92–102.

A play for eighteen characters; suitable for geometry classes. Dialogue clever and brisk; properties and costumes simple but effective.

Shea, John. "Mr. Chips Teaches Geometry." *School Science and Mathematics*, 1940, 40: 720–726.

Short humorous skit on locus concept; nine characters, no properties.

Skerrett, Josephine. "A Mathematical Nightmare." MATHEMATICS TEACHER, 1929, 22: 413–417.

A play written by a high school pupil, showing what would happen if there were no mathematics; ten players; for junior high school.

Slaught, H. E. "The Evolution of Numbers: An Historical Drama in Two Acts." MATHEMATICS TEACHER, 1928, 21: 305–315; *American Mathematical Monthly*, 1928, 35: 146–151.

Play based on the history of the number

system of algebra; instructive as well as entertaining. Calls for an audience of some mathematical maturity.

Smith, Alice. "The Case of Matthew Mattix." MATHEMATICS TEACHER, 1933, 26: 286–291.

A comedy in two scenes; requires seventeen speaking parts.

Smith, Alice. "Snow White and the Seven Dwarves." MATHEMATICS TEACHER, 1944, 37: 27–30.

A light skit in three short acts; calls for eleven players. Appropriate for junior or senior high school.

Snyder, Ruth. "If." MATHEMATICS TEACHER, 1929, 22: 482–486.

A play in two acts showing the importance of mathematics; nine players.

Sperks, Marie. "The Determination of Taxes in the Community." *School Science and Mathematics*, 1942, 42: 454–462.

Can be tied in readily with a unit on taxation and with related projects. Requires ten speaking parts; suitable for junior high school.

Stark, Marion. "Modern Mathematics Looks Up Its Ancestors." *American Mathematical Monthly*, 1936, 43: 299–304.

Costume play on the history of mathematics; blank verse; ten characters. Stimulating as well as instructive.

Ward, J. A. "A Wax Works Show." MATHEMATICS TEACHER, 1941, 34: 266–268.

A brief comedy skit on Newton, Einstein and Archimedes.

Whelan, A. Marie. "It Can't Happen Here." *American Mathematical Monthly*, 1938, 45: 617–628.

A clever and amusing musical farce, based on the theme: Why study mathematics in college? In two acts and five scenes; six principal players, and chorus.

Whitaker, Helen. "The Mathematical Quest." *School Science and Mathematics*, 1920, 20: 457–459; MATHEMATICS TEACHER, 1925, 18: 356–358.

A short mathematics pageant, in verse; one act. Requires fifty characters, but only two speaking parts. Suitable for senior high school.

Woodard, Mary. "A Study in Human Stupidity." MATHEMATICS TEACHER, 1945, 38: 362–367.

A biographical play on the life of Evariste Galois; short, but compelling; musical accompaniment.

Woolery, Freda. "Much Ado About Mathematics." MATHEMATICS TEACHER, 1945, 38: 23–35.

Portrays the life of a person who has been forbidden to use mathematics; five scenes and an epilogue. Suitable for junior high school.

B. PROGRAMS FOR ASSEMBLIES AND MATHEMATICS CLUBS

Adler, Irving. "Fun with Mathematics: An Assembly Program." MATHEMATICS TEACHER, 1949, 42: 153–155.

Adler, Irving. "Theory and Practice." MATHEMATICS TEACHER, 1948, 41: 218–220.

Bogen, Isidore. "Mathematics in Life." *High Points*, 1949, 31: 73–79.

Braverman, Benjamin. "The Quiz in a Mathematics Assembly Program." *High Points*, 1947, 29: 64–69.

Fisher, Genevieve. "A Mathematical Assembly Program." *Bulletin, Kansas Association of Mathematics Teachers*, 1945, 20: 5–7.

Gulden, M. "Mathematics Club Program." MATHEMATICS TEACHER, 1924, 17: 350–358.

Hatcher, Frances. "A Living Theorem; a Class Day Program." *School Science and Mathematics*, 1916, 16: 39–40.

Hoag, R. "Sources of Program Material and Some Types of Program Work Which Might Be Undertaken by High School Mathematics Clubs." MATHEMATICS TEACHER, 1931, 24: 492–502.

Klotz, C. E. "Mathematics Clubs for High Schools." *School Activities*, 1949, 21: 59–61.

Lehmann, Paul. "A Math Circus; an Assembly Program." *School Science and Mathematics*, 1947, 47: 560–563.

Parsons, G. "Work of a Junior Mathematical Association." *Mathematical Gazette*, 1935, 19: 65–72.

Person, R. "Junior High School Mathematics Clubs." MATHEMATICS TEACHER, 1941, 34: 228–229.

Phillips, K. "Junior High School Mathematics Club." *High School Journal*, 1930, 13: 68–71.

Porterfield, Jacob. "Fun for the Mathematics Club." MATHEMATICS TEACHER, 1944, 37: 354–357.

Reed, Z. "High School Mathematics Clubs." MATHEMATICS TEACHER, 1925, 18: 341–363.

Sullivan, O. A. "The High School Mathematics Club." MATHEMATICS TEACHER, 1942, 35: 275–276.

"Thirty Topics for a Mathematics Program." *School Science and Mathematics*, 1927, 27: 170–171.

Vaughn, Adah. "Professor Whiz and His Class in Math. Magic." *School Science and Mathematics*, 1939, 39: 540–545.

Weiss, M. "Math Club—Streamlined." *High Points*, 1939, 21: 74–77.

Wheeler, A. "Mathematics Club Program." MATHEMATICS TEACHER, 1923, 16: 385–390.

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